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K-BANHATTI AND K-HYPER BANHATTI INDICES OF DOMINATING DAVID DERIVED NETWORK

WEI GAO, BATSHA MUZAFFAR, WAQAS NAZEER[1]

Abstract. Let G be connected graph with vertex $V(G)$ and edge set $E(G)$. The first and second K-Banhatti indices of G are defined as $B_1(G) = \sum_{ue}[d_G(u) + d_G(e)]$ and $B_2(G) = \sum_{ue}[d_G(u)d_G(e)]$,where ue means that the vertex u and edge e are incident in G. The first and second K-hyper Banhatti indices of G are defined as $HB_1(G) = \sum_{ue}[d_G(u) + d_G(e)]^2$ and $HB_2(G) = \sum_{ue}[d_G(u)d_G(e)]^2$. In this paper, we compute the first and second K-Banhatti and K-hyper Banhatti indices of Dominating David Derived networks.

AMS Mathematics Subject Classification: 05C05, 05C07, 05C35.
Key words and phrases: K-Banhatti indices; K-hyper Banhatti indices; dominating David Derived networks.

1. Introduction

Chemical graph theory is a branch of graph theory in which a chemical compound is represented by simple graph called molecular graph in which vertices are atoms of compound and edges are the atomic bounds. A graph is connected if there is atleast one connection between its vertices. Throughout this paper we take G a connected graph. If a graph does not contain any loop or multiple edges then it is called a network. Between two vertices u and v, the distance is the shortest path between them and is denoted by in graph G. For a vertex v of G the *degree* is number of vertices attached with it. The edge connecting the vertices u and v will be denoted by uv. Let $d_G(e)$ denote the degree of an edge e in G, which is defined by $d_G(e) = d_G(u)+d_G(v)-2$ with $e = uv$. The degree and

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[1] Corresponding Author

valence in chemistry are closely related with each other. We refer the book [1] for more details. Now a day another emerging field is Cheminformatics, which helps to predict biological activities with the relationship of Structure-property and quantitative structure-activity. Topological indices and physico-chemical properties are used in prediction of bioactivity if underlined compounds are used in these studies [2, 3].
A number that describe the topology of a graph is called topological index. In 1947, the first and most studied topological index was introduced by Weiner [4]. More details about this index can be found in [5, 6]. In 1975, Milan Randić introduced the Randić index [7].
Bollobas *et al.* [8] and Amic *et al.* [9] in 1998, working independently defined the generalized Randić index. This index was studied by both mathematicians and chemists [10]. For details about topological indices, we refer [11, 12].
The first and second K-Banhatti indices of G are defined as:

$$B_1(G) = \sum_{ue} [d_G(u) + d_G(e)]$$

and

$$B_2(G) = \sum_{ue} [d_G(u) \times d_G(e)],$$

where ue means that the vertex u and edge e are incident in G. The first and second K-hyper Banhatti indices of G are defined as:

$$HB_1(G) = \sum_{ue} [d_G(u) + d_G(e)]^2$$

and

$$HB_2(G) = \sum_{ue} [d_G(u) \times d_G(e)]^2.$$

We refer [13] for details about these indices.
The David derived and dominating David derived network of dimension n can be constructed as follows [14]: consider an n dimensional star of David network, insert a new vertex on each edge and split it into two parts, we will get David derived network $DD(n)$ of dimension n.

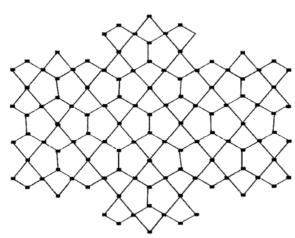

Figure 1. Dominating David derived network of the first type $D_1(2)$

The dominating David derived network of the first type of dimension n which can be obtained by connecting vertices of degree 2 of $DDD(n)$ by an edge that are not in the boundary and is denoted by $D_1(n)$ [14].

The dominating David derived network of the second type of dimension n can be obtained by subdividing the new edges in $D_1(n)$ [14] and is denoted by $D_1(2)$.

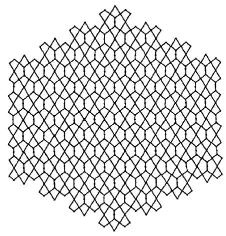

Figure 2. Dominating David derived network of the second type $D_2(2)$

The dominating David derived network of the second type of dimension n denoted by $D_3(n)$ can be obtained from $D_1(n)$ by introducing a parallel path of length 2 between the vertices of degree two that are not in the boundary [14, 15].

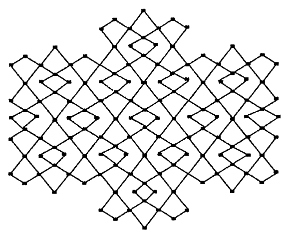

Figure 3. Dominating David derived network of the third type $D_3(2)$

In this article, we compute first and second K-Banhatti index and first and second hyper K-Banhatti index of Dominating David derived networks of first, second and third type. Throughout this paper $E_{m,n} = \{e = uv \in E(G); d_u = m, d_v = n\}$ and $|E_{m,n}(G)|$ is the number of elements in $E_{m,n}(G)$.

2. Main Results

In this section, we present our main results.

Theorem 2.1. *Let $G = D_1(n)$ be the dominating David derived network of 1^{st} type. Then the first and the second K-Banhatti indices of $D_1(n)$ are*

$$\begin{aligned} B_1[D_1(n)] &= 1485n^2 + 1624n - 1002, \\ B_2[D_1(n)] &= 3204n^2 + 764n - 3292. \end{aligned}$$

Proof. Let $G = D_1(n)$ be the dominating David derived network of 1^{st} type. From Figure 1, the edge partition of dominating David derived network of 1^{st} type $D_1(n)$ based on degrees of end vertices of each edge is give in Table 1. First K-Banhatti index of $D_1(n)$ is calculated as

$$\begin{aligned} B_1[D_1(n)] &= \sum_{ue} [d_G(u) + d_G(e)] \\ &= \sum_{ue \in E_{2,2}} [(d_G(u) + d_G(e)) + (d_G(v) + d_G(e))] \end{aligned}$$

TABLE 1. Edge partition of Dominating David derived network of first type

$(d_u, d_v); e = uv \in E(G)$	Number of edges	Degree of Edges $d_G(e) = d_G(u) + d_G(v) - 2$
$(2,2)$	$4n$	2
$(2,3)$	$4n-4$	3
$(2,4)$	$28n-16$	4
$(3,3)$	$9n^2-13n+24$	4
$(3,4)$	$36n^2-56n+24$	5
$(4,4)$	$36n^2-56n+20$	6

$$
\begin{aligned}
&+ \sum_{ue \in E_{2,3}} [(d_G(u) + d_G(e)) + (d_G(v) + d_G(e))] \\
&+ \sum_{ue \in E_{2,4}} [(d_G(u) + d_G(e)) + (d_G(v) + d_G(e))] \\
&+ \sum_{ue \in E_{3,3}} [(d_G(u) + d_G(e)) + (d_G(v) + d_G(e))] \\
&+ \sum_{ue \in E_{3,4}} [(d_G(u) + d_G(e)) + (d_G(v) + d_G(e))] \\
&+ \sum_{ue \in E_{4,4}} [(d_G(u) + d_G(e)) + (d_G(v) + d_G(e))] \\
= \; & 4n[(2+2)+(2+2)] + (4n-4)[(2+3)+(3+3)] \\
&+(28n-16)[(2+4)+(4+4)] \\
&+(9n^2-13n+24)[(3+4)+(3+4)] \\
&+(36n^2-56n+24)[(3+5)+(4+5)] \\
&+(36n^2-56n+20)[(4+6)+(4+6)] \\
= \; & 1458n^2 + 1624n - 1002.
\end{aligned}
$$

Second K-Banhatti index of $D_1(n)$ is calculated as

$$
\begin{aligned}
B_2[D_1(n)] &= \sum_{ue} [d_G(u) d_G(v)] \\
&= \sum_{ue \in E_{2,2}} [(d_G(u) d_G(e)) + (d_G(v) d_G(e))] \\
&\quad + \sum_{ue \in E_{2,3}} [(d_G(u) d_G(e)) + (d_G(v) d_G(e))] \\
&\quad + \sum_{ue \in E_{2,4}} [(d_G(u) d_G(e)) + (d_G(v) d_G(e))]
\end{aligned}
$$

$$
\begin{aligned}
&+\sum_{ue\in E_{3,3}}[(d_G(u)d_G(e))+(d_G(v)d_G(e))]\\
&+\sum_{ue\in E_{3,4}}[(d_G(u)d_G(e))+(d_G(v)d_G(e))]\\
&+\sum_{ue\in E_{4,4}}[(d_G(u)d_G(e))+(d_G(v)d_G(e))]\\
=\;&4n[(2+2)+(2.2)]+(4n-4)[(2+3)+(3.3)]\\
&+(28n-16)[(2.4)+(4.4)]\\
&+(9n^2-13n+24)[(3.4)+(4.4)]\\
&+(36n^2-56n+24)[(3.5)+(4.5)]\\
&+(36n^2-56n+20)[(4.6)+(4.6)]\\
=\;&3204n^2+764n-3292.
\end{aligned}
$$

□

Theorem 2.2. *Let $G = D_1(n)$ be the dominating David derived network of 1^{st} type. Then the first and the second K-hyper Banhatti indices of $D_1(n)$ are*

$$
\begin{aligned}
HB_1[D_1(n)] &= 13302n^2-16623n+6146,\\
HB_2[D_1(n)] &= 66564n^2-89092n+33892.
\end{aligned}
$$

Proof. Let $G = D_1(n)$ be the dominating David derived network of 1^{st} type. Then first K-hyper Banhatti index of $D_1(n)$ is calculated as

$$
\begin{aligned}
HB_1[D_1(n)] &= \sum_{ue}[d_G(u)+d_G(v)]^2\\
&= \sum_{ue\in E_{2,2}}[(d_G(u)+d_G(e))^2+(d_G(v)+d_G(e))^2]\\
&\quad+\sum_{ue\in E_{2,3}}[(d_G(u)+d_G(e))^2+(d_G(v)+d_G(e))^2]\\
&\quad+\sum_{ue\in E_{2,4}}[(d_G(u)+d_G(e))^2+(d_G(v)+d_G(e))^2]\\
&\quad+\sum_{ue\in E_{3,3}}[(d_G(u)+d_G(e))^2+(d_G(v)+d_G(e))^2]\\
&\quad+\sum_{ue\in E_{3,4}}[(d_G(u)+d_G(e))^2+(d_G(v)+d_G(e))^2]\\
&\quad+\sum_{ue\in E_{4,4}}[(d_G(u)+d_G(e))^2+(d_G(v)+d_G(e))^2]\\
&= 4n[(2+2)^2+(2+2)^2]+(4n-4)[(2+3)^2+(3+3)^2]\\
&\quad+(28n-16)[(2+4)^2+(4+4)^2]
\end{aligned}
$$

$$
\begin{aligned}
&+(9n^2-13n+24)[(3+4)^2+(3+4)^2] \\
&+(36n^2-56n+24)[(3+5)^2+(4+5)^2] \\
&+(36n^2-56n+20)[(4+6)^2+(4+6)^2] \\
= \;& 13302n^2-16623n+6146.
\end{aligned}
$$

Second K-hyper Banhatti index of $D_1(n)$ is calculated as

$$
\begin{aligned}
HB_2[D_1(n)] &= \sum_{ue}[d_G(u)d_G(v)]^2 \\
&= \sum_{ue\in E_{2,2}}[(d_G(u)d_G(e))^2+(d_G(v)d_G(e))^2] \\
&\quad+\sum_{ue\in E_{2,3}}[(d_G(u)d_G(e))^2+(d_G(v)d_G(e))^2] \\
&\quad+\sum_{ue\in E_{2,4}}[(d_G(u)d_G(e))^2+(d_G(v)d_G(e))^2] \\
&\quad+\sum_{ue\in E_{3,3}}[(d_G(u)d_G(e))^2+(d_G(v)d_G(e))^2] \\
&\quad+\sum_{ue\in E_{3,4}}[(d_G(u)d_G(e))^2+(d_G(v)d_G(e))^2] \\
&\quad+\sum_{ue\in E_{4,4}}[(d_G(u)d_G(e))^2+(d_G(v)d_G(e))^2] \\
&= 4n[4^2+4^2]+(4n-4)[6^2+9^2] \\
&\quad+(28n-16)[8^2+16^2] \\
&\quad+(9n^2-13n+24)[12^2+12^2] \\
&\quad+(36n^2-56n+24)[15^2+20^2] \\
&\quad+(36n^2-56n+20)[24^2+24^2] \\
&= 66564n^2-89092n+33892.
\end{aligned}
$$

□

Theorem 2.3. *Let $G = D_2(n)$ be the dominating David derived network of 2^{nd} type. Then the first and the second K-Banhatti indices of $D_2(n)$ are*

$$
\begin{aligned}
B_1[D_2(n)] &= 1530n^2-1810n+650, \\
B_2[D_2(n)] &= 32584n^2-4127n+1506.
\end{aligned}
$$

Proof. Let $G = D_2(n)$ be the dominating David derived network of 2^{nd} type. Table 2 shows the edge partition of dominating David derived network of 2^{nd} type $D_2(n)$ based on degrees of end vertices of each edge
First K-Banhatti index of $D_2(n)$ is calculated as

$$
B_1[D_2(n)] = \sum_{ue}[d_G(u)+d_G(v)]
$$

TABLE 2. Edge partition of Dominating David Derived Network of second type

$(d_u, d_v); e = uv \in E(G)$	Number of edges	Degree of Edges $d_G(e) = d_G(u) + d_G(v) - 2$
$(2, 2)$	$4n$	2
$(2, 3)$	$18n^2 - 22n + 6$	3
$(2, 4)$	$28n - 16$	4
$(3, 4)$	$36n^6 - 56n + 24$	5
(4,4)	$36n^6 - 56n + 20$	6

$$\begin{aligned}
&= \sum_{ue \in E_{2,2}} [(d_G(u) + d_G(e)) + (d_G(v) + d_G(e))] \\
&\quad + \sum_{ue \in E_{2,3}} [(d_G(u) + d_G(e)) + (d_G(v) + d_G(e))] \\
&\quad + \sum_{ue \in E_{2,4}} [(d_G(u) + d_G(e)) + (d_G(v) + d_G(e))] \\
&\quad + \sum_{ue \in E_{3,4}} [(d_G(u) + d_G(e)) + (d_G(v) + d_G(e))] \\
&\quad + \sum_{ue \in E_{4,4}} [(d_G(u) + d_G(e)) + (d_G(v) + d_G(e))] \\
&= 4n[(2+2) + (2+2)] + (18n^2 - 22n + 6)[(2+3) + (3+3)] \\
&\quad +(28n - 16)[(2+4) + (4+4)] \\
&\quad +(36n^6 - 56n + 24)[(3+5) + (4+5)] \\
&\quad +(36n^6 - 56n + 20)[(4+6) + (4+6)] \\
&= 1530n^2 - 1810n + 650.
\end{aligned}$$

Second K-Banhatti index of $D_2(n)$ is calculated as

$$\begin{aligned}
B_1[D_2(n)] &= \sum_{ue} [d_G(u) d_G(v)] \\
&= \sum_{ue \in E_{2,2}} [(d_G(u) d_G(e)) + (d_G(v) d_G(e))] \\
&\quad + \sum_{ue \in E_{2,3}} [(d_G(u) d_G(e)) + (d_G(v) d_G(e))] \\
&\quad + \sum_{ue \in E_{2,4}} [(d_G(u) d_G(e)) + (d_G(v) d_G(e))] \\
&\quad + \sum_{ue \in E_{3,4}} [(d_G(u) d_G(e)) + (d_G(v) d_G(e))]
\end{aligned}$$

$$
\begin{aligned}
& + \sum_{ue\in E_{4,4}} [(d_G(u)d_G(e)) + (d_G(v)d_G(e))] \\
= \; & 4n[(2.2)+(2.2)] + (18n^2-22n+6)[(2.3)+(3.3)] \\
& +(28n-16)[(2.4)+(4.4)] \\
& +(36n^6-56n+24)[(3.5)+(4.5)] \\
& +(36n^6-56n+20)[(4.6)+(4.6)] \\
= \; & 32584n^2-4127n+1506.
\end{aligned}
$$

□

Theorem 2.4. *Let $G = D_2(n)$ be the dominating David derived network of 2^{nd} type. Then the first and the second K-hyper Banhatti indices of $D_2(n)$ are*

$$
\begin{aligned}
HB_1[D_1(n)] &= 1351n^2-1693n+6246, \\
HB_2[D_1(n)] &= 22606n^2-28486n+10582.
\end{aligned}
$$

Proof. Let $G = D_2(n)$ be the dominating David derived network of 2^{nd} type. First K-hyper Banhatti index of $D_2(n)$ is calculated as

$$
\begin{aligned}
HB_1[D_2(n)] &= \sum_{ue} [d_G(u)+d_G(v)]^2 \\
&= \sum_{ue\in E_{2,2}} [(d_G(u)+d_G(e))^2 + (d_G(v)+d_G(e))^2] \\
&\quad + \sum_{ue\in E_{2,3}} [(d_G(u)+d_G(e))^2 + (d_G(v)+d_G(e))^2] \\
&\quad + \sum_{ue\in E_{2,4}} [(d_G(u)+d_G(e))^2 + (d_G(v)+d_G(e))^2] \\
&\quad + \sum_{ue\in E_{3,4}} [(d_G(u)+d_G(e))^2 + (d_G(v)+d_G(e))^2] \\
&\quad + \sum_{ue\in E_{4,4}} [(d_G(u)+d_G(e))^2 + (d_G(v)+d_G(e))^2] \\
&= 4n[(2+2)^2+(2+2)^2] + (18n^2-22n+6)[(2+3)^2+(3+3)^2] \\
&\quad +(28n-16)[(2+4)^2+(4+4)^2] \\
&\quad +(36n^6-56n+24)[(3+5)^2+(4+5)^2] \\
&\quad +(36n^6-56n+20)[(4+6)^2+(4+6)^2] \\
&= 1351n^2-1693n+6246.
\end{aligned}
$$

Second K-hyper Banhatti index of $D_2(n)$ is calculated as

$$
HB_2[D_2(n)] = \sum_{ue} [d_G(u)d_G(v)]^2
$$

$$
\begin{aligned}
&= \sum_{ue\in E_{2,2}} [(d_G(u)d_G(e))^2 + (d_G(v)d_G(e))^2] \\
&\quad + \sum_{ue\in E_{2,3}} [(d_G(u)d_G(e))^2 + (d_G(v)d_G(e))^2] \\
&\quad + \sum_{ue\in E_{2,4}} [(d_G(u)d_G(e))^2 + (d_G(v)d_G(e))^2] \\
&\quad + \sum_{ue\in E_{3,4}} [(d_G(u)d_G(e))^2 + (d_G(v)d_G(e))^2] \\
&\quad + \sum_{ue\in E_{4,4}} [(d_G(u)d_G(e^2) + (d_G(v)d_G(e))^2] \\
&= 4n[(2.2)^2 + (2.2)^2] + (18n^2 - 22n + 6)[(2.3)^2 + (3.3)^2] \\
&\quad +(28n - 16)[(2.4)^2 + (4.4)^2] \\
&\quad +(36n^6 - 56n + 24)[(3.5)^2 + (4.5)^2] \\
&\quad +(36n^6 - 56n + 20)[(4.6)^2 + (4.6)^2] \\
&= 22606n^2 - 28486n + 10582.
\end{aligned}
$$

□

Theorem 2.5. *Let $G = D_3(n)$ be the dominating David derived network of 3^{rd} type. Then the first and the second K-Banhatti indices of $D_3(n)$ are*

$$
\begin{aligned}
B_1[D_3(n)] &= 1944n^2 - 2128n + 600, \\
B_2[D_3(n)] &= 4320n^2 - 8224n + 2112.
\end{aligned}
$$

Proof. Let $G = D_3(n)$ be the dominating David derived network of 3^{rd} type. Table 3 shows the edge partition of dominating David derived network of 3^{rd} type $D_3(n)$ based on degrees of end vertices of each edge. First K-Banhatti index of $D_3(n)$ is calculated as

TABLE 3. Edge partition of Dominating David derived network of third type

$(d_u, d_v); e = uv \in E(G)$	Number of edges	Degree of Edges $d_G(e) = d_G(u) + d_G(v) - 2$
$(2, 2)$	$4n$	2
$(2, 4)$	$36n^2 - 20n$	4
$(4, 4)$	$72n^2 - 108n + 44$	6

$$
\begin{aligned}
B_1[D_3(n)] &= \sum_{ue} [d_G(u) + d_G(v)] \\
&= \sum_{ue\in E_{2,2}} [(d_G(u) + d_G(e)) + (d_G(v) + d_G(e))]
\end{aligned}
$$

$$
\begin{aligned}
&+\sum_{ue\in E_{2,4}}[(d_G(u)+d_G(e))+(d_G(v)+d_G(e))]\\
&+\sum_{ue\in E_{4,4}}[(d_G(u)+d_G(e))+(d_G(v)+d_G(e))]\\
=\;& 4n[(2+2)+(2+2)]+(36n^2-20n)[(2+4)+(4+4)]\\
&+(72n^2-108n+44)[(2+6)+(4+6)]\\
=\;& 1944n^2-2128n+600.
\end{aligned}
$$

Second K-Banhatti index is calculated as

$$
\begin{aligned}
B_2[D_3(n)] &= \sum_{ue}[d_G(u)+d_G(v)]\\
&= \sum_{ue\in E_{2,2}}[(d_G(u)d_G(e))+(d_G(v)d_G(e))]\\
&\quad+\sum_{ue\in E_{2,4}}[(d_G(u)d_G(e))+(d_G(v)d_G(e))]\\
&\quad+\sum_{ue\in E_{4,4}}[(d_G(u)d_G(e))+(d_G(v)d_G(e))]\\
&= 4n[(2.2)+(2.2)]+(36n^2-20n)[(2.4)+(4.4)]\\
&\quad+(72n^2-108n+44)[(2.6)+(4.6)]\\
&= 4320n^2-8224n+2112.
\end{aligned}
$$

□

Theorem 2.6. *Let $G = D_3(n)$ be the dominating David derived network of 3^{rd} type. Then the first and the second K-hyper Banhatti indices of $D_3(n)$ are*

$$
\begin{aligned}
HB_1[D_3(n)] &= 18000n^2-23472n+8800,\\
HB_2[D_3(n)] &= 94464n^2-130688n+50688.
\end{aligned}
$$

Proof. Let $G = D_3(n)$ be the dominating David derived network of 3^{rd} type. Then the first K-hyper Banhatti index is calculated as

$$
\begin{aligned}
HB_1[D_3(n)] &= \sum_{ue}[d_G(u)+d_G(v)]^2\\
&= \sum_{ue\in E_{2,2}}[(d_G(u)+d_G(e))^2+(d_G(v)+d_G(e))^2]\\
&\quad+\sum_{ue\in E_{2,4}}[(d_G(u)+d_G(e))^2+(d_G(v)+d_G(e))^2]\\
&\quad+\sum_{ue\in E_{4,4}}[(d_G(u)+d_G(e))^2+(d_G(v)+d_G(e))^2]\\
&= 4n[(2+2)^2+(2+2)^2]+(36n^2-20n)[(2+4)^2+(4+4)^2]\\
&\quad+(72n^2-108n+44)[(2+6)^2+(4+6)^2]
\end{aligned}
$$

$$= \quad 18000n^2 - 23472n + 8800.$$

Second K-hyper Banhatti index of $D_3(n)$ is calculated as

$$\begin{aligned}
HB_1[D_3(n)] &= \sum_{ue}[d_G(u) + d_G(v)]^2 \\
&= \sum_{ue \in E_{2,2}} [(d_G(u)d_G(e))^2 + (d_G(v)d_G(e))^2] \\
&\quad + \sum_{ue \in E_{2,4}} [(d_G(u)d_G(e))^2 + (d_G(v)d_G(e))^2] \\
&\quad + \sum_{ue \in E_{4,4}} [(d_G(u)d_G(e))^2 + (d_G(v)d_G(e))^2] \\
&= 4n[(2.2)^2 + (2.2)^2] + (36n^2 - 20n)[(2.4)^2 + (4.4)^2] \\
&\quad +(72n^2 - 108n + 44)[(2.6)^2 + (4.6)^2] \\
&= 94464n^2 - 130688n + 50688.
\end{aligned}$$

□

3. Conclusion

In the present report, we have computed first and second K-Banhatti and K-hyer Banhatti indices of Dominating David derived networks of first, second and third type.

Competing Interests

The author(s) do not have any competing interests in the manuscript.

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Wei Gao
School of Information Science and Technology, Yunnan Normal University, Kunming 650500, China.
e-mail: gaowei@ynnu.edu.cn

Batsha Muzaffar
Department of Mathematics and Statistics, University of Lahore Lahore 54590, Pakistan.
e-mail: batshamuzaffar41@gmail.com

Waqas Nazeer
Division of Science and Technology, University of Education, Lahore-54000, Pakistan.
e-mail: nazeer.waqas@ue.edu.pk