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On Making an Informed Choice between Two Lomax-based Continuous Probability Distributions Using Lifetime Data

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Authors' contributions

This work was carried out in collaboration between all authors. All authors read and approved the final manuscript.

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Abstract

Probability distributions and their generalisations have contributed greatly in the modeling and analysis of random variables. However, due to the increased introduction of new distributions there has been a major problem with choosing and applying the right distribution for a given set of data. In most cases, it is discovered that the data set in question fits two or more probability distributions and hence one must be chosen among others. The Lomax-Weibull and Lomax-Log-Logistic distributions introduced in an earlier study using a Lomax-based generator were found to be positively skewed and may be victims of this situation especially when modelling positively skewed datasets. In this article, we apply the two distributions to some selected datasets to compare their performance and provide useful insight on how to select the most fit among them when dealing with a real-life situation. We used the log-likelihood value, AIC, CAIC, BIC, HQIC, Cramér-Von Mises (W*) and Anderson Darling (A*) statistics as performance evaluation tools for selecting between the two distributions.

Keywords: Lomax-Weibull distribution; Lomax-Log-Logistic distribution; Lomax-based generator; performance evaluation.

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1 Introduction

Recently, researchers have developed compound probability distributions which have been proven to have better performance than the well-known standard probability distributions. These studies are meant to introduce a higher level of skewness in the existing probability distributions by extending on a well-known distribution or link function under some facts and assumptions. Further work on some of these studies stated above have led to the production of some compound probability distributions such as skew normal distribution by Azzalini [1], the generalised Weibull distribution by Mudholkar and Kollia [2], the exponentiated Weibull distribution by Mudholkar et al. [3], the beta-Weibull distribution by Famoye et al. [4], the Kumaraswamy normal distribution by Cordeiro and De Castro [5], the Lomax-Frechet distribution by Gupta et al. [6] and the Lomax-Gumbel distribution by Gupta et al. [7] etc. all of which have been proven to be better than their parent or baseline counterparts.

Making a clear choice between two related probability distribution functions is very vital and has been done by some researchers under the following topics and considerations; "a test of discriminating between models" by Atkinson [8], "discrimination between the Log-normal and Weibull distribution" by Dumonceaux and Antle [9], "a method for discriminating between models with discussion" by [10], "discrimination between the Log-normal and Gamma distribution" by Kundu and Manglick [11], "On Modeling of Lifetimes Data Using Exponential and Lindley Distributions" by Shanker et al. [12], "A Study of Probability Models in Monitoring Environmental Pollution in Nigeria" by Oguntunde et al. [13] as well as "discriminating between the Weibull-normal and the generalised Weibull-normal distributions" by Ieren and Yahaya [14].

Cordeiro et al. [15] proposed a Lomax generator with two extra positive parameters for extending continuous distributions and in their study, some distributions like the Lomax-normal, Lomax–Weibull, Lomax-log-logistic and Lomax–Pareto distributions were studied. The properties of the generator including ordinary and incomplete moments, quantile function, moment generating function, mean and median deviations, distribution of the order statistics and some entropy measures were also presented. They discussed the estimation and inference of the parameters via the method of maximum likelihood with a minification process based on the marginal Lomax-exponential distribution. The point of interest and attraction for the authors here is to compare the performance of the Lomax-Weibull distribution to that of the Lomax-Log-logistic distribution because of the following reasons: (i) both distributions have the same shape pattern and are skewed to the right according to the graphical representation of the two distributions by Cordeiro et al. [15]; (ii) applications to three real life datasets show that the Lomax-Weibull distribution is better than beta-Weibull, Kummaraswamy-Weibull, Lomax-exponential, beta-pareto, Weibull and Burr distributions among others, however, its performance has not been compared to that of the Lomax-Log-logistic distribution which seems to be related to the Lomax-Weibull distribution by graphical observations. Therefore, the aim of this paper is to compare the fitness of the Lomax-Weibull distribution to that of the Lomax-Log-logistic distribution defined by Cordeiro et al. [15] using some statistical measures and seven real life datasets.

The rest of this article is presented as follows: in Section 2 we state the definition of Lomax distribution, Lomax-G family, Lomax-Weibull and Lomax-Log-logistic distributions as well as some statistics and goodness-of-fit measures. In section 3, we present some datasets, their summary and analysis and discussions. Finally, some concluding remarks are being provided in section 4.

2 Materials and Methods

2.1 The Lomax Distribution and Lomax-G family of distributions

The Lomax distribution was formed to handle analysis of business failure data by Lomax [16]. The distribution is useful for a wide range application such as income and wealth inequality, size of towns, actuarial studies, medical and biological sciences, engineering, lifetime and reliability modelling.

The probability density function (*pdf*) of the Lomax random variable *X* with parameters *α and β* is given by

$$f(x)=\frac{\alpha}{\beta}\left[1+\left(\frac{x}{\beta}\right)\right]^{-(\alpha+1)} \quad (2.1),$$

and the corresponding cumulative distribution function (*cdf*) is given as

$$F(x)=1-\left[1+\left(\frac{x}{\beta}\right)\right]^{-\alpha} \quad (2.2)$$

where $x>0,\alpha>0,\beta>0$, while *α and β* are the shape and scale parameters respectively.

According to Cordeiro et al. [15], the *cdf* and *pdf* of the Lomax-G family distributions (based on a Lomax generator) are respectively given by:

$$F(x)=1-\left\{\frac{\beta}{\beta-\log[1-G(x)]}\right\}^{\alpha} \quad (2.3)$$

And

$$f(x)=\alpha\beta^{\alpha}\frac{g(x)}{[1-G(x)]\{\beta-\log[1-G(x)]\}^{\alpha+1}}, \quad (2.4)$$

where g(*x*) and G(*x*) are the *pdf* and *cdf* of any continuous distribution to be extended, while $\alpha>0$ and $\beta>0$ are the additional new parameters responsible for the scale and shape of the distribution respectively.

2.2 The Lomax-Weibull distribution (*LWD*)

The *cdf* and *pdf* of a random variable *X* taking a Weibull distribution with scale parameter *a*>0 and shape parameter *b*>0 are respectively given by (2.5) and (2.6):

$$G(x)=1-e^{-ax^{b}} \quad (2.5)$$

$$g(x)=abx^{b-1}e^{-ax^{b}} \quad (2.6)$$

where $x>0,a>0,b>0$ where *a and b* are the scale and shape parameters respectively.

By substituting equations (2.5) and (2.6) into (2.3) and (2.4) and solving, we get the *cdf* and *pdf* of the Lomax-Weibull distribution respectively as:

$$F(x)=1-\frac{\beta^{\alpha}}{\left[\beta-\log\left[1-\left(1-e^{-ax^{b}}\right)\right]\right]^{\alpha}} \quad (2.7)$$

$$f(x)=\frac{\alpha\beta^{\alpha}abx^{b-1}}{\left(\beta-\log\left[1-\left(1-e^{-ax^{b}}\right)\right]\right)^{\alpha+1}} \tag{2.8}$$

A plot of the *pdf* of the LWD for varing parameters is as follows.

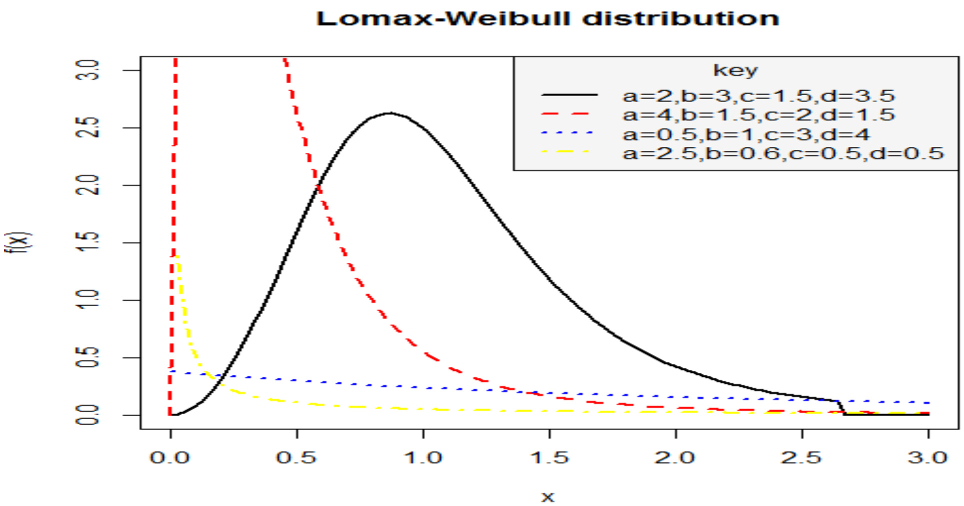

Fig. 2.1. A plot of *pdf* of the LWD for Varying parameter values given $c=\alpha$ *and* $d=\beta$.

Considering the plot above, we can rightly say that the *LWD* is skewed to the right with a very high degree of peakedness and could also exhibit other shapes depending on the parameter values which are useful for modeling real life data sets.

2.3 The Lomax-Log-Logistic Distribution (*LLD*)

The *cdf* and *pdf* of the Log-logistic distribution are respectively given by:

$$G(x)=1-\left[1+\left(\frac{x}{a}\right)^{b}\right]^{-1} \tag{2.9}$$

And

$$g(x)=\frac{b}{a^{b}}x^{b-1}\left[1+\left(\frac{x}{a}\right)^{b}\right]^{-2} \tag{2.10}$$

For $x>0$, where $a>0$ and $b>0$ are the scale and shape parameters respectively.

By substituting equations (2.9) and (2.10) into (2.3) and (2.4) and solving, we get the *cdf* and *pdf* of the Lomax-Log-logistic distribution as follows:

$$F(x)=1-\frac{\beta^{\alpha}}{\left[\beta+\log\left[1+\left(\frac{x}{a}\right)^{b}\right]\right]^{\alpha}} \quad (2.11)$$

$$f(x)=\frac{\alpha\beta^{\alpha}b}{a^{b}}x^{b-1}\frac{\left[1+\left(\frac{x}{a}\right)^{b}\right]^{-1}}{\left(\beta+\log\left[1+\left(\frac{x}{a}\right)^{b}\right]\right)^{\alpha+1}} \quad (2.12)$$

Below is a graph of the *pdf* of LLD for varying values of the model parameters.

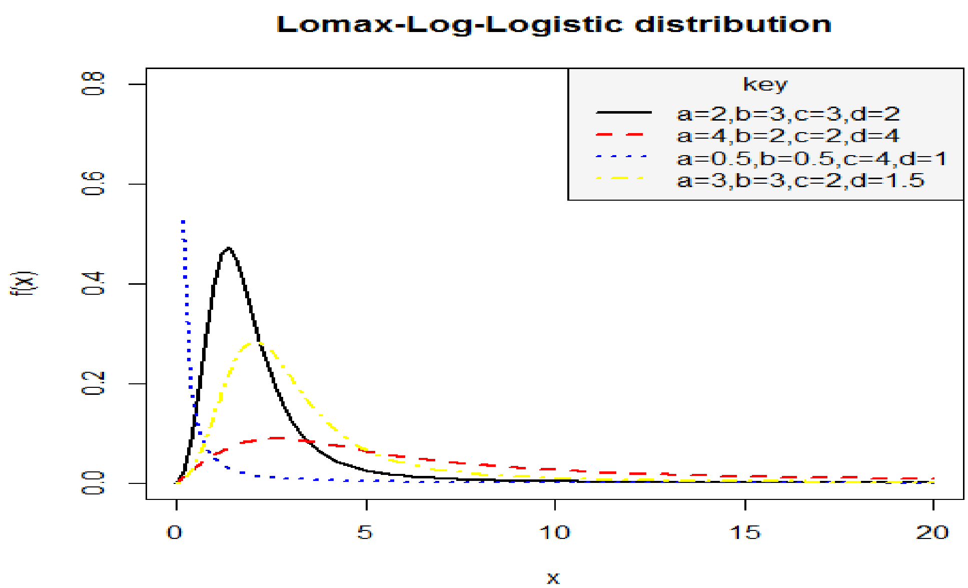

Fig. 2.2. A plot of *pdf* of the LLD for arbitrary parameter values given $c=\alpha-and-d=\beta$.

The plot for the *pdf* above indicates that the LLD is positively skewed and can take various shapes good for modeling lifetime datasets.

2.4 Goodness-of-Fit test

To compare these two distributions, the following information criteria are used, namely: the natural logarithm of the likelihood function value (ll), Akaike Information Criterion (AIC), Consistent Akaike Information Criterion (CAIC), Bayesian Information Criterion (BIC), and Hannan Quin Information Criterion (HQIC). These statistics are given as:

$AIC=-2ll+2P$; $BIC=-2ll+P\log(n)$, ; $CAIC=-2ll+\frac{2Pn}{(n-P-1)}$ and $HQIC=-2ll+2P\log\left[\log(n)\right]$

where ll denotes the natural logarithm of the likelihood function evaluated at the *MLEs*, *P* is the number of parameters in the distribution and *n* is the size of the sample used.

We also used goodness-of-fit tests in order to know which distribution fits the data better, we apply the Cramér-Von Mises *(W*)*, and Anderson Darling *(A*)* test statistics. Additional information about these statistics can be obtained from [17]. These statistics can be computed as:

$$W^* = w^2\left(1 + 0.5/n\right) \text{ and}$$

$$A^* = A^2\left(1 + 0.75/n + 2.25/n^2\right)$$

where

$$w^2 = \sum_{i=1}^{n}\left\{u_i - (2_i - 1)/(2n)\right\}^2 + 1/(12n),$$

$$A^2 = -n - \frac{1}{n}\sum_{i=1}^{n}\left\{(2_i - 1)\log(u_i) + (2n + 1 - 2_i)\log(1 - u_i)\right\},$$

$V_i = F\left(x_i, \hat{\theta}\right)$ is the known *cdf* with $\hat{\theta}$ (a k-dimensional parameter vector), $y_i = \Phi^{-1}\left(V_i\right)$ is the standard quantile function, $u_i = \Phi\left\{\left(y_i - \bar{y}/s_y\right)\right\}$, $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ and $s_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(y_i - \bar{y}\right)^2$.

Note: When choosing between the two distributions, the distribution with the smaller measures for these criteria shall be considered as the best to fit the data.

3 Results and Discussion

3.1 Analysis of data

In this section, seven different datasets were used to fit both the *LWD* and Lomax-Log-Logistic distribution by applying the formulas of the test statistics in section 4 to discriminating between the two mentioned distributions. The available data sets and their respective summary statistics are provided in as follows;

Dataset I: This dataset stands for the remission times of a random sample of 128 bladder cancer patients. It has been used by Lee and Wang [18]. It is summarised as follows:

Table 3.1. Summary statistics for dataset I

Parameters	*n*	Minimum	Q_1	Median	Q_3	Mean	Maximum	Variance	Skewness	Kurtosis
Values	128	0.0800	3.348	6.395	11.840	9.366	79.05	110.425	3.3257	19.1537

Dataset II: This dataset is the strength data of glass of the aircraft window reported by Fuller et al. [19].

Table 3.2. Summary statistics for dataset II

Parameters	n	Minimum	Q_1	Median	Q_3	Mean	Maximum	Variance	Skewness	Kurtosis
Values	31	18.83	25.51	29.90	35.83	30.81	45.38	52.61	0.43	2.38

Dataset III: This dataset stands for the waiting times before service of 100 Bank customers and examined and analysed by Ghitany et al. [20] for fitting the Lindley distribution.

Table 3.3. Summary statistics for dataset III

Parameters	n	Minimum	Q_1	Median	Q_3	Mean	Maximum	Variance	Skewness	Kurtosis
Values	100	0.80	4.675	8.10	13.020	9.877	38.500	52.3741	1.4953	5.7345

Dataset IV: This dataset represents the lifetime's data relating to relief times (in minutes) of 20 patients receiving an analgesic and reported by Gross and Clark [21] and has been used by Shanker et al. [22].

Table 3.4. Summary statistics for dataset IV

Parameters	n	Minimum	Q_1	Median	Q_3	Mean	Maximum	Variance	Skewness	Kurtosis
Values	20	1.10	1.475	1.70	2.05	1.90	4.10	0.4958	1.8625	7.1854

Dataset V: This data represent the survival times in weeks for male rats from [23].

Table 3.5. Summary statistics for dataset V

Parameters	n	Minimum	Q_1	Median	Q_3	Mean	Maximum	Variance	Skewness	Kurtosis
Values	20	40.00	86.75	119.00	140.80	113.45	165.00	1280.892	-0.3552	2.2120

Dataset VI: The dataset is from [24]. The data given arose in tests on endurance of deep groove ball bearings. The data are the number of million revolutions before failure for each of the 23 ball bearings in the life tests. Its summary is given as follows:

Table 3.6. Summary statistics for dataset VI

Parameters	n	Minimum	Q_1	Median	Q_3	Mean	Maximum	Variance	Skewness	Kurtosis
Values	23	17.88	47.20	67.80	95.88	72.23	173.40	1404.78	1.0089	3.9288

Dataset VII: This dataset represents 66 observations of the breaking stress of carbon fibres of 50mm length (in GPa) given by Nichols and Padgett [25]. The descriptive statistics for this data are as follows:

Table 3.7. Descriptive statistics for dataset VII

Parameters	n	Minimum	Q_1	Median	Q_3	Mean	Maximum	Variance	Skewness	Kurtosis
Values	66	0.390	2.178	2.835	3.278	2.760	4.900	0.795	-0.1285	3.2230

From the summary statistics of the seven data sets, we found that data sets I, II, III, IV and VI are positively skewed, while V is approximately normal. Also, data sets I, III and IV have higher kurtosis while others have moderate level of peakness.

Table 3.8. Performance of the distributions using their *AIC*, *CAIC*, *BIC* and *HQIC* values of the models' *MLEs* based on datasets I-VII

Datasets	*Models*	*Log-likelihood value*	*Parameter Estimates*	*Statistics*	*Model Ranks*
Dataset I	*LWD*	420.7675	$\hat{a}$=0.3928 $\hat{b}$=0.8735 $\hat{\alpha}$=4.4202 $\hat{\beta}$=6.5906	*AIC=849.5355* *CAIC=849.8607* *BIC=860.9437* *HQIC=854.1707*	*2*
	LLD	411.4727	$\hat{a}$=7.9519 $\hat{b}$=1.6252 $\hat{\alpha}$=8.1254 $\hat{\beta}$=5.4517	*AIC=830.9454* *CAIC=831.2707* *BIC=842.3536* *HQIC=835.5806*	*1*
Dataset II	*LWD*	146.435	$\hat{a}$=0.0987 $\hat{b}$=0.7832 $\hat{\alpha}$=7.1911 $\hat{\beta}$=5.3806	*AIC=300.8701* *CAIC=302.4085* *BIC=306.606* *HQIC=302.7398*	*1*
	LLD	148.548	$\hat{a}$=9.5745 $\hat{b}$=3.3012 $\hat{\alpha}$=2.2311 $\hat{\beta}$=6.2759	*AIC=305.096* *CAIC=306.6345* *BIC=310.832* *HQIC306.9658*	*2*
Dataset III	*LWD*	342.2547	$\hat{a}$=0.5010 $\hat{b}$=0.7455 $\hat{\alpha}$=3.4439 $\hat{\beta}$=8.6494	*AIC=692.5095* *CAIC=692.9305* *BIC=702.9302* *HQIC=696.7269*	*2*
	LLD	319.8772	$\hat{a}$=9.5864 $\hat{b}$=2.2868 $\hat{\alpha}$=7.5884 $\hat{\beta}$=4.8861	*AIC=647.7543* *CAIC=648.1754* *BIC=658.175* *HQIC=651.9718*	*1*
Dataset IV	*LWD*	10.3037	$\hat{a}$=4.0707 $\hat{b}$=1.5688 $\hat{\alpha}$=0.9416 $\hat{\beta}$=2.9579	*AIC=28.6075* *CAIC=31.2741* *BIC=32.5904* *HQIC=29.3849*	*1*
	LLD	15.7405	$\hat{a}$=1.6082 $\hat{b}$=7.7819 $\hat{\alpha}$=5.2092 $\hat{\beta}$=7.0803	*AIC=39.4809* *CAIC=42.1476* *BIC=43.4639* *HQIC=40.2585*	*2*
Dataset V	*LWD*	132.1458	$\hat{a}$=0.4511 $\hat{b}$=0.8217 $\hat{\alpha}$=1.2186 $\hat{\beta}$=6.5257	*AIC=272.2916* *CAIC=274.9582* *BIC=276.2745* *HQIC=273.0691*	*1*
	LLD	138,5343	$\hat{a}$=7.3688 $\hat{b}$=1.9915 $\hat{\alpha}$=1.5932 $\hat{\beta}$=9.0109	*AIC=285.0687* *CAIC=287.7354* *BIC=289.0516* *HQIC=285.8462*	*2*
Dataset VI	*LWD*	128.6364	$\hat{a}$=0.2197 $\hat{b}$=0.3748 $\hat{\alpha}$=6.8929 $\hat{\beta}$=4.2691	*AIC=265.2728* *CAIC=267.495* *BIC=269.8148* *HQIC=266.4151*	*1*
	LLD	138.7535	$\hat{a}$=8.8356 $\hat{b}$=2.1849	*AIC=285.507* *CAIC=287.7292*	*2*

Datasets	*Models*	*Log-likelihood value*	*Parameter Estimates*	*Statistics*	*Model Ranks*
			$\hat{\alpha}$=2.6049	*BIC=290.0489*	
			$\hat{\beta}$=9.4333	*HQIC=286.6492*	
Dataset VII	*LWD*	83.5572	$\hat{a}$=2.9920	*AIC=175.1145*	*1*
			$\hat{b}$=1.5482	*CAIC=175.7702*	
			$\hat{\alpha}$=1.3201	*BIC=183.8731*	
			$\hat{\beta}$=7.2416	*HQIC=178.5754*	
	LLD	86.7655	$\hat{a}$=6.9929	*AIC=181.531*	*2*
			$\hat{b}$=3.5394	*CAIC=182.1867*	
			$\hat{\alpha}$=9.2142	*BIC=190.2896*	
			$\hat{\beta}$=0.4701	*HQIC=184.9917*	

Table 3.8 shows parameter MLEs to each one of the two fitted distributions for the seven data sets (Datasets I-VII), the table also shows the relative values of *ll*, *AIC*, *BIC*, *CAIC and HQIC* for each model. The values in Table 3.8 show that the *LWD* performs better for five datasets while the *LLD* performs better for just two datasets. We also notice that the five datasets for which the *LWD* performs better than *LLD* are those with low degree of kurtosis and the two datasets for which the *LLD* performs better are the ones with higher degree of kurtosis. Hence, we can say at this point that the *LWD* should be used for modeling positively skewed datasets most especially those with moderate or low kurtosis while the *LLD* should be applied when the datasets are skewed to the right with higher degree of peakedness.

Table 3.9. Performance of the distributions using the *W and *A** values of the models based on dataset I, III, V and VI**

Datasets	**Models**	***W****	***A****	**Model Ranks**
Dataset I	*LWD*	0.0312	0.2084	*1*
	LLD	0.0382	0.2856	*2*
Dataset III	*LWD*	0.0212	0.1663	*1*
	LLD	0.0536	0.3742	*2*
Dataset V	*LWD*	0.0872	0.5949	*1*
	LLD	0.1136	0.7662	*2*
Dataset VI	*LWD*	0.0302	0.1867	*1*
	LLD	0.0521	0.3937	*2*

Table 3.9 displays the values of goodness-of-fit statistics *W** and *A** for the two distributions under four selected datasets (I, III, V and VI). The results from table 3.9 confirm that irrespective of the coefficient of kurtosis, the *LWD* performs better than the *LLD*. Based on the values of these statistics in table 3.9, we can confidently say that the *LWD* is better than the *LLD* and hence should be used for analysing positively skewed datasets. Hence, the statement above is in line with [15] who also said that the *LWD* is better than the Beta-Weibull, Kummaraswamy-Weibull, Weibull and the Burr distributions.

4 Conclusion

In this article, a comparison has been made between two Lomax-based continuous probability distributions namely; the *LWD* and *LLD*. We considered seven real life data sets of different status and used the value of the log-likelihood function, *AIC*, *CAIC*, *BIC*, *HQIC*, Cramér-Von Mises *(W*)* and Anderson Darling *(A*)* statistics as performance measures for selecting between these two distributions. Our analysis and results proved that the *LWD* has better performance compared to the *LLD* irrespective of the level of skewness and kurtosis.

Competing Interests

Authors have declared that no competing interests exist.

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