



HYPERGRAPH COLORING BASED ALGORITHM FOR CHANNEL ALLOCATION

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ABSTRACT

The channel allocation plays an important role in the network performance. The unavailability of channels at certain instant poses challenges for spectrum allocation process in Cognitive Radio Networks. The opportunistic use of the spectrum by the secondary user in an efficient manner proves of good use for effective utilisation of the spectrum which further improves the communication in Cognitive Radio Networks. We have evaluated the performance of hypergraph coloring algorithm in accordance to various parameters taken into consideration like Average Sum Rate (ASR) to see the variation with respect to the number of channels and the number of users. We propose Hypergraph Coloring Algorithm which has better performance than the graph-based channel allocation algorithm. The existing algorithm approach is giving better result in terms of utilization of spectrum in an efficient manner involving Hypergraph coloring algorithm for channel allocation.

Keywords: Channel allocation; hypergraph coloring; cognitive radio networks; secondary users; signal to interference plus noise Ratio.

1. INTRODUCTION

Hypergraph coloring is a channel assignment process that involves coloring the vertices which is equivalent to the assignment of channels. It involves the interference in the form of edges such as independent and cumulative interferer and making sure that the same channel is not allocated to certain particular link twice hence playing a crucial role in avoiding the interference. The approach of Hypergraph coloring is appropriate to avoid the interference. In most of the previous approaches the communication quality of

link was affected by the interference from strong cumulative interferer to weak cumulative interferer. The proposed approach varies from the traditional graph approach in the way that in traditional graph the edge used to be between maximum two nodes but the hyperedge is an edge which can contain as many nodes as possible, see [1,2]. The approach followed in this process mainly involves Signal to Interference plus Noise Ratio (SINR) which enables us to check whether a particular cognitive user who is to be allocated to certain subcarrier is fit for it or not. The allocation is done [3] keeping in mind the Hypergraph

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coloring and the concept that the nodes contained in the Hyperedge all belongs to distinct colors. The concurrency is assured by assigning the subcarriers to the secondary users by using the Orthogonal Frequency Division Multiple Access (OFDMA) method. Average Sum Rate is calculated for optimizing the channel allocation process in order to ensure proper and efficient utilization of the spectrum, see [4, 5, 6].

2. RELATED WORK

Junior et al. in [7] proposed the technique which involves channel assignment algorithm that tends to be distributive in nature. The extent of knowledge about the neighbors involving a fully distributed approach aims at tremendously reducing the computational costs. Moreover, the number of channels that are required for channel allocation reduces. This is a technique which is adaptive in nature and the communication overhead incurred too is low. Azari et al. in [8] proposed a method for performance characterization in terms of per user degree of freedom (DoF) involving clustered cooperative beamforming. The cellular networks were used to represent clusters of cooperative base stations by using edge caching. The optimal use of backhaul resources and usage of cache in an efficient manner enables us to maintain qualitative insights about cellular networks using edge caching. It aims at finding the optimal non-convex complex problems. Zhnag et al. [9] proposed hypergraph-based resource allocation for the purpose of cross cell device to device communication. It emphasized on ensuring the proper communication even in the case of reduced cell sizes and to cope up with the problem of communication or link failure over long range. The Hypergraph technique is used to represent this model where vertices or nodes are used as cellular users and hyperedges denotes the mutual interference that occurs due to the various cellular users. It aims at utilization of spectrum in an efficient manner involving Hypergraph coloring algorithm for channel allocation which mainly involves two phases. Firstly, it aims at hypergraph construction by computing the independent and cumulative interferer and comparing the SINR value that need to be taken into consideration while completing the process of Hypergraph formation. Finally, after the hypergraph construction the second phase involves partitioning of different channels into different clusters that help the primary and secondary users to tune accordingly to the channel that is allocated irrespective of the frequencies that tend to be there in the neighboring environment. It also aimed at optimizing the sum rate while comparing its variation with respect to the number of users and channels. Sun et al. in [10] used the Hypergraph based method involving the

interference metric to formulate the problem occurring during channel access. The designing to achieve the Potential Nash Equilibrium is done to obtain the optimal level by the use of distributed learning algorithm. The aim is to minimize the interference metric in order to ensure smooth communication and less effect of interference. The spectrum efficiency is improved and the proposed hypergraph algorithm outperforms the existing schemes. An algorithm was proposed by Wang et al. in [11] to solve the problem which arose due to the ultra-low latency and ultra-high data rate especially in the case of communication over 5G networks. The technique to be used for future 5G systems most probably can be non-orthogonal multiple access mainly because of its ability for reducing the latency in data transmission and improving the data efficiency. It emphasizes on the spatial reuse for V2X (Vehicle to everything) networks which are (Non-Orthogonal Multiple Access) NOMA-integrated considering the interference scenario which is complicated. Mohan and Kaushal [12] proposed an algorithm for path allocation specially implemented on optical networks. This scheme is based on Dynamic routing and wavelength assignment for efficient traffic grooming. Aizaz et al. [13] resolved the issue of dynamic channel allocation by applying a hybrid technique of simulated annealing and genetic algorithm. It started by using an initial set of constraints and further led to successive generations of solutions.

3. PROPOSED MODEL AND ASSUMPTIONS

The wireless network which involves the interference cannot be modelled efficiently by the edges which contain only two nodes because there may arise a case where link quality may be affected when some weak interferer combined together form a strong cumulative interferer. Thus, in contrast to the graph approach followed traditionally we now lay emphasis on hypergraph approach.

A Cognitive Radio Network (CRN) involves secondary users which are to be assigned certain channels keeping in mind the interference which can be modelled efficiently in the terminologies related to a hypergraph $H = (X, E)$ where " $X = \{x_1, x_2, x_3, \dots, x_n\}$ denotes a finite set of nodes, a hypergraph H on X is a family $E = \{e_1, e_2, e_3, \dots, e_m\}$ of subsets of X " such that

$$e_z \neq \phi \{z = 1, 2, 3, \dots, m\},$$

where n and m denotes the number of nodes and hyperedges respectively.

$$\bigcup_{z=1}^m e_z = X$$

The incidence matrix illustrated below in Fig. 1 consists of rows which corresponds to nodes and columns that denotes the hyperedges.

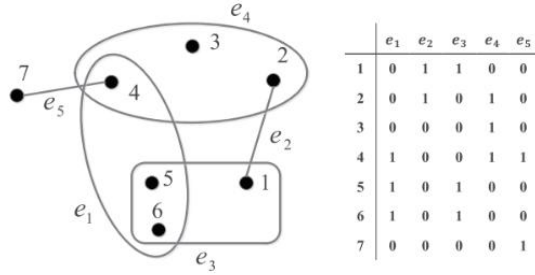


Fig. 1. Incidence matrix corresponding to following hypergraph

The initial phase of channel allocation involves comparing the SINR value of SU (Secondary User) at each subcarrier. In addition to this the secondary users which needs to be assigned to respective sub carriers is done using hypergraph coloring. It is necessary to mention that the nodes which are in the same hyperedge cannot be assigned the same color. Hyperedge consists of one or more nodes unlike the traditional graph in which the number of nodes are limited to exactly two.

- A hyperedge represents the Independent interferer or Cumulative interferer.
- Conflict exists if the nodes that are included in the hyperedge is assigned the same color.

A. Assumptions

- The communication range equals the interference range.
- The nodes which form the hyperedge must not be colored by the sane color.
- OFDMA (Orthogonal Frequency Division Multiple Access) ensures multiple access by assigning the secondary users to corresponding subcarriers.

B. Conditions and Parameters

The allocation model consists of Channel Allocation matrix along with several conditions which plays a crucial role in determining the independent and cumulative interferer. The various matrices and parameters are as follows:

1. **Allocation Matrix :** The channel Allocation matrix

as defined in [14] is denoted by

$$S_{M \times K} = (A^{M \times K})$$

where $A_{M \times K} = [\alpha_{m,k}]$ represents channel allocation matrix for secondary user

$1 \leq m \leq M$ M= Number of secondary users
 $1 \leq k \leq K$ K= Number of channels

$$\alpha_{m,k} = \begin{cases} 1, & \text{when Channel is allocated to SU} \\ 0, & \text{otherwise} \end{cases}$$

$\alpha_{m,k}$: denotes the value for m^{th} secondary user and k^{th} channel

2. **Conditions and Parameters:** The conditions for independent interferer threshold

$$\frac{g_m^{SU}}{g_n^{SU}} < \delta_{SU} \tag{1}$$

$$\frac{P^{SU} g_m^{SU}}{P^{PU} g_n^{PU}} < \delta_{PU} \tag{2}$$

The conditions for cumulative Interferer threshold

$$\frac{P^{SU} g_m^{SU}}{\sum_{m=1}^M P^{SU} g_m^{SU} + \sum_{n=1}^N P^{PU} g_n^{PU}} < \eta_{SU} \tag{3}$$

where

$1 \leq m \leq M$ M= Number of Secondary users
 $1 \leq n \leq N$ N= Number of Primary users
 $1 \leq k \leq K$ K= Number of Channels

In the above equations(1) – (3)

P : denotes the transmission power
 g_r : it denotes the channel gain of the r^{th} user
 δ_c : denotes the threshold selected to determine the severity of interference of c.
 η_c : denotes the interference threshold to verify if they become interferers if cumulated.

The conditions which shows that channel can be allocated only to PU or SU at a particular time is given by

$$\sum_{m=1}^M \alpha_{m,k} \leq 1 \tag{4}$$

$\alpha_{m,k}$: defines channel assignment variable for m_{th} secondary user and k_{th} channel

C. Proposed Algorithm - Hypergraph Coloring

1. Average Sum Rate Calculation

SINR(γ) is defined as the Signal-to-Interference plus Noise ratio which is calculated for each subcarrier that is formed by splitting of Single channel into various

subchannels.

The SINR for user i on subcarrier x , such that $x \in 1, \dots, N$ is given by

$$\gamma_i^x = \frac{P_i^x \cdot h_{ii}^x}{n_0 + \sum_{j \neq i} P_j^x \cdot h_{ji}^x}$$

where

P_i^x : is transmission power of user i on subcarrier x
 h_{ji} : corresponds the channel gain to user i receiver from user j on subcarrier x
 n_0 : corresponds to Noise Power which satisfies the gaussian distribution with 0 mean and variance σ^2

Rate of Secondary User is denoted as follows

$$R_{SU} = \sum_{n=1}^N \log_2(1 + \gamma_m) \cdot \alpha_{m,k}$$

For efficient utilization of the Spectrum we need to optimize the channel allocation variable $[\alpha_{m,k}]$ Average Sum Rate is therefore denoted by

$$A_{SR} = \text{Max} \sum_{k=1}^K \sum_{m=1}^M \log_2(1 + \gamma_m) \cdot \alpha_{m,k}$$

2. Algorithm Description

The Algorithm is divided into two phases:

- (a) Initially in the Channel allocation phase we take into account the hypergraph construction. This involves the comparison of SINR with some threshold values. The construction takes place in two steps, firstly by forming the edges with the help of independent Interferer and secondly forming a hyperedge by taking into account cumulative Interferer [15].
- (b) In the hypergraph coloring phase we take into consideration the vertex with minimum mono-degree. It needs to be mentioned that strongly delete the vertex from induced hypergraph or sub-hypergraph. Then, it will be followed by finding the vertex randomly in the color set until there is no vertex in the color set. In this way the Hypergraph Coloring takes place [16].

Algorithm 1 Hypergraph Construction

1: Inputs: "N" Number of Secondary users and "K" number of channels
2: /*Independent Interferer Phase*/
3: while Untill all Secondary users find their independent interferer **do**
4: for Each Secondary user **do**
5: if

$$\frac{g_m^{SU}}{g_n^{SU}} < \delta_{SU}$$

or

$$\frac{p^{SU} g_m^{SU}}{p^{PU} g_n^{PU}} < \delta_{PU}$$

then

6: Secondary user is an independent interferer

7: end if

8: end for

9: Form edges with the independent interferer

10: end while

11: /*Cumulative Interferer Phase*/

12: while Untill all Secondary users find their Cumulative interferer **do**

13: for Each Secondary user **do**

14: if

$$\frac{p^{SU} g_m^{SU}}{\sum_{m=1}^M p^{SU} g_m^{SU} + \sum_{n=1}^N p^{PU} g_n^{PU}} < \eta_{SU}$$

then

15: Secondary user is an Cumulative interferer

16: end if

17: end for

18: Form hyperedges with the Cumulative interferer

19: end while

20: The vertex which is not in any hyperedge or edge forms hyperedge itself.

Algorithm 2 Hypergraph Colouring

- 1: $i = n$, $H_n = H$. Find a vertex of minimum monodegree in H_n and label it X_n .
- 2: while $i > 0$ do
- 3: Set $i = i - 1$, and strongly delete the vertex X_{i+1} and form an induced sub-hypergraph $H_i = H_{i+1} - X_{i+1}$
- 4: Find the vertex of minimum monodegree in H_i and label it X_i
- 5: end while
- 6: Successively, starting from $i=1$, Color the node X_i in a color selected randomly from the color set that is available correspondingly.
- 7: When the available color set is empty, make the node X_i uncolored.

4. SIMULATION MODEL AND RESULTS

The ultimate aim is to derive results and get some information about the system hence analyzing the results by running the simulation. The following concepts which are to be taken into consideration while doing the simulation runs is organized by MATLAB.

Model - The executable files are called (M - files) and are often called .m files. The experimentation process involves the invariant code that is if the code is changed then it will account to different model.

Study - It involves a sequence of events to get some insight of one or more model. Mostly one performs certain set of experiment which proves useful and from which the conclusion can be inferred.

Experiment - It mainly consists of exploring the model by taking into account its parameter space. It involves many measurements with certain simulation runs having similar parameters but differ in the seeds.

Replication - It mainly signifies repetition. A replication is mainly denoted and characterized by the seed values it accounts upon.

Run - Run can be characterized by the processing of simulation which leads in giving us the insight of network model which is taken into consideration. Furthermore, the refinement of the measurements is done after noting the simulation parameters and specification of model.

A. Simulation Parameters

The following performance values are considered: Average Sum Rate with number of channels and number of secondary users, SINR ratio with Number of Subcarriers. Here comparison on the basis of changing parameters is drawn and implementation of simulation is done on varying constraints. The simulation parameters given in the TABLE I have been inherited from [17].

Table 1. Parameters for simulation

Transmission Power P_{SU}	13 dBm
Transmission Power P_{PU}	23 dBm
Carrier Frequency	2.3 GHz
Transmission Bandwidth	20 MHz
Noise Figure	5 dB
Threshold $\delta_{PU} = \eta_{PU}$	25 dB
Threshold $\delta_{SU} = \eta_{SU}$	20 dB
Path Loss Model	UMi in

B. Performance Metrics

Two important performance metrics are represented in our experiments:

1. **SINR ratio:** SINR denotes signal-to-interference plus noise ratio which is calculated for each subcarrier that is formed by splitting of Single channel into various subchannels..
2. **Average Sum Rate :** It is defined as the summation of rate of SU over the entire range of all the channels.

The first metric is very important to draw the comparison amongst various users as it sets a certain upper bound which helps us to determine whether the certain user has to be taken into consideration for a particular sub carrier or not. High SINR gives it a greater chance or priority in respect to other users. Average Sum Rate plays a crucial role in determining as to check whether the certain sequence of steps lead us to the efficient utilization of the spectrum by utilizing it in a rather opportunistic manner. Based on the above system settings of the network the simulation is performed and results are laid before in the next section.

C. Simulation Results

The simulation took place using MATLAB simulation mathematical tool. The above mentioned performance metrics are used for the sole purpose of comparison and to keep a check on its variation with respect to certain simulation parameters.

1. SINR Ratio

(a) **By varying the number of subcarriers**

In Fig. 2, when the number of subcarrier increases the SINR ratio initially increases rapidly but slowly reaches a steady or saturation state and this value starts from a higher range in case where cyclic prefix is 2 as compared to when it is 0. The CP acts as a guard band and helps us to send data on separate channels simultaneously without much of interference hence leading to the greater range of SINR value [18]. This signifies that more the SINR value more will be

its priority amongst secondary users. Moreover, the use of cyclic prefix provides robustness and reduces inter-symbol interference.

2. Average Sum Rate

(a) **By varying the number of channels**

The Average Sum Rate first increases linearly with the increase in the number of channels but after a certain increase in the number of channels it starts declining minimally as illustrated in Fig. 3.

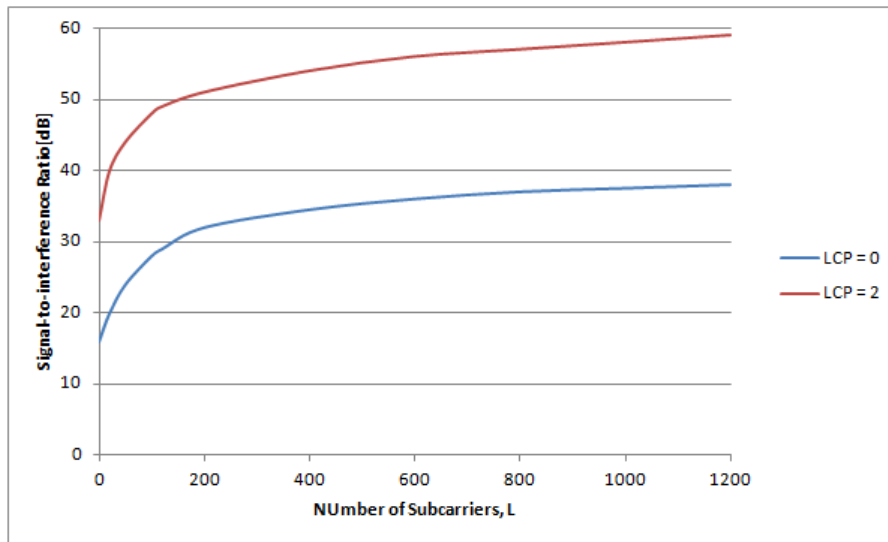


Fig. 2. Comparison of SINR ratio with Number of Sub-carriers

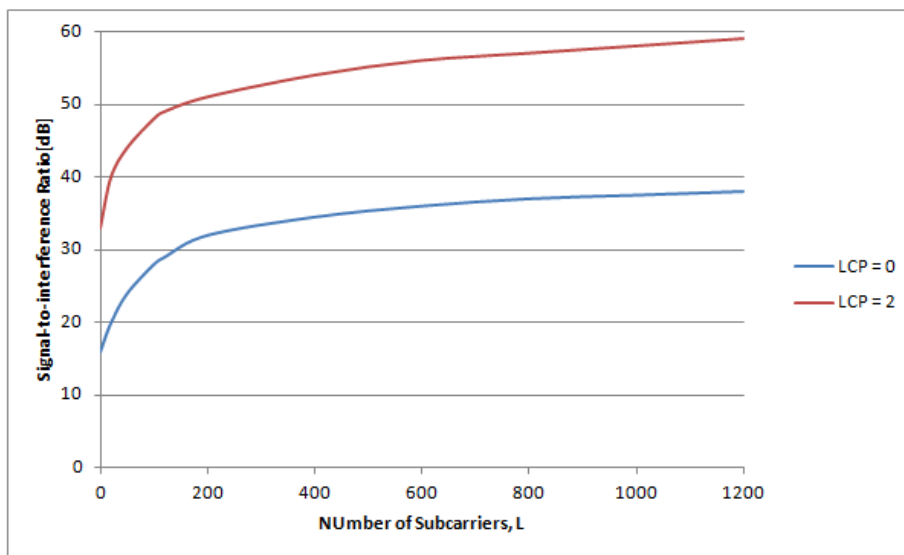


Fig. 3. Comparison of Average Sum Rate with Number of Channels for Hypergraph based Channel allocation

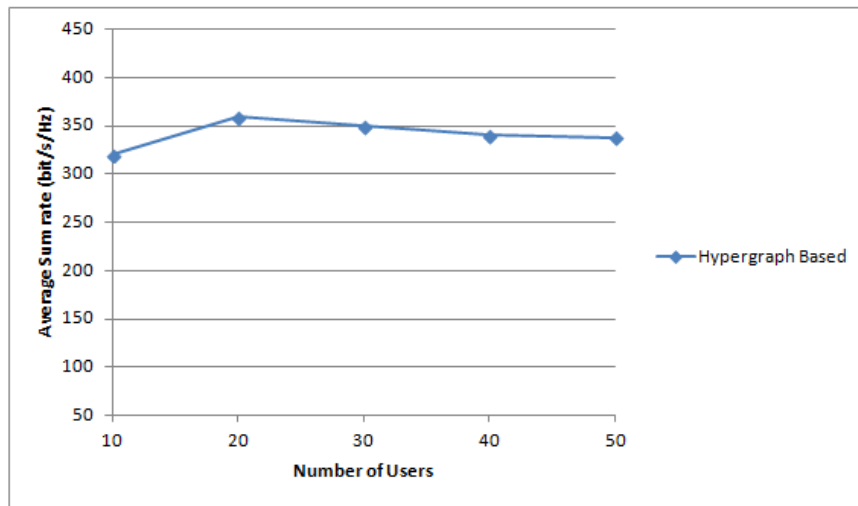


Fig. 4. Comparison of Average Sum Rate with Number of Users for Hypergraph based Channel allocation

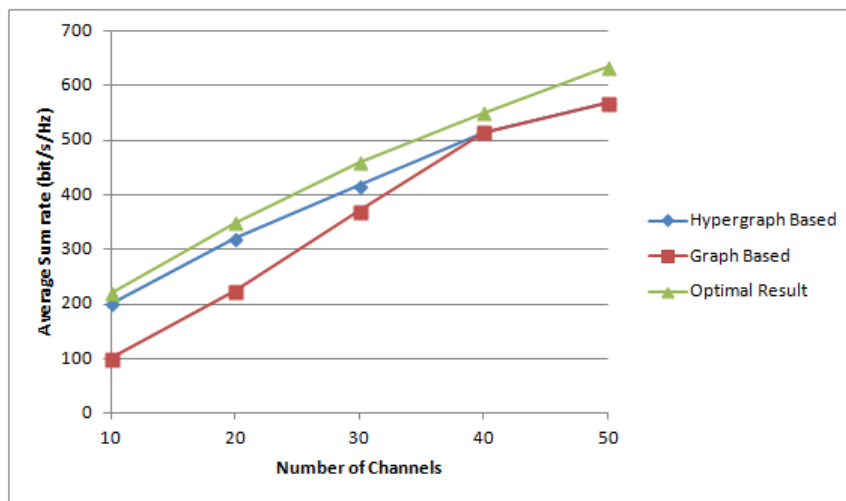


Fig. 5. Comparison between Hypergraph based, Graph based and Optimal result variation of Average Sum Rate with respect to Number of Channels

(b) By varying the number of users

In Fig. 4, Average Sum Rate first increases linearly with the small increase in the number of users but after a certain increase in the number of channels it starts declining before achieving the state of saturation.

Hence, observing its change with respect to optimal result. It shows that initially upto a certain number channels hypergraph based allocation does well in comparison to graph based but, later after further increase in number of channels the variation shown by hypergraph and graph based is similar which shows minimal decline in both cases.

(c) Comparison between different approaches of channel allocation of Average Sum Rate with respect to Number of Channels

A comparison of variation in Average Sum Rate with respect to Number of channels for Hypergraph based and graph based channel allocation is drawn in Fig. 5.

(d) Comparison between different approaches of channel allocation of Average Sum rate with respect to number of users

A comparison of variation in Average Sum Rate with respect to Number of users for Hypergraph based and Graph based channel allocation [19,20] is drawn in

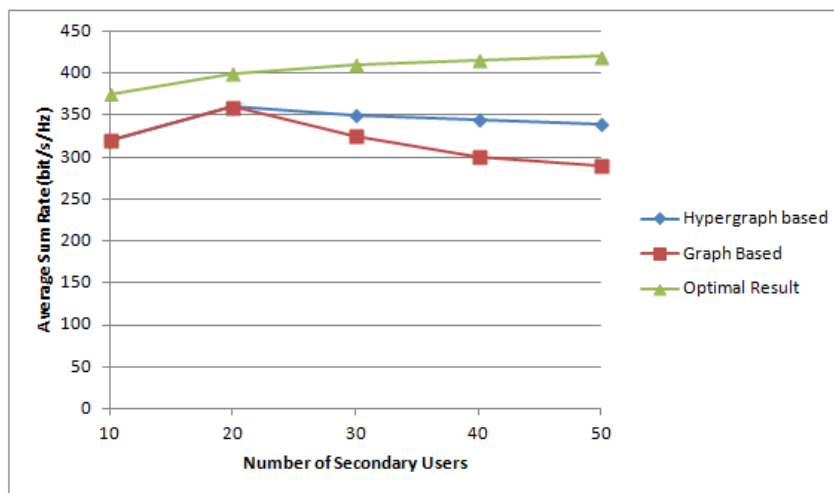


Fig. 6. Comparison between Hypergraph based, Graph based and Optimal result variation of Average Sum Rate with respect to Number of Users

Fig. 6. Hence, observing its change with respect to optimal result [21]. It shows that initially upto a certain number of users the trend for hypergraph based allocation rises linearly and similar to graph based but, later after further increase in number of users the variation shown by trend of hypergraph shows a linear decline before reaching the state of saturation but performs well in comparison to graph based.

5. CONCLUSION

We have used the MATLAB tool for the simulation of the proposed algorithm. The proposed hypergraph-based channel allocation scheme is compared with the graph based channel allocation algorithm. Initially the comparison of SINR ratio is done with respect to the number of carriers followed by the variation of Average Sum Rate with respect to the number of secondary users and number of channels. We have shown that the performance of the Hypergraph based channel allocation is better than the graph based channel allocation. Furthermore, the performance of the proposed scheme increases with the number of channels.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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