British Journal of Mathematics & Computer Science
3(3): 341-351, 2013

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Are Near Earth Objects the Key to Optimization Theory

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Short Communication

Received: 24 February 2013
Accepted: 27 April 2013
Published: 07 May 2013

Abstract

This note suggests that near earth objects and Central Force Optimization have something in common, that NEO theory may hold the key to solving some vexing problems in deterministic optimization: local trapping and proof of convergence. CFO analogizes Newton's laws to locate the global maxima of a function. The NEO-CFO nexus is the striking similarity between CFO's D_{avg} and an NEO's ΔV curves. Both exhibit oscillatory plateau-like regions connected by jumps, suggesting that CFO's metaphorical "gravity" indeed behaves like real gravity, thereby connecting NEOs and CFO and being the basis for speculating that NEO theory may address difficult issues in optimization.

Keywords: *Central force optimization, CFO, near earth objects, NEO, optimization theory, proof of convergence, local trapping, metaheuristic, evolutionary algorithm*.

1 Introduction

This note suggests that the theory of gravitationally trapped Near Earth Objects (NEOs) provides an analytical framework for the further theoretical development of Central Force Optimization (CFO). NEO theory may lead to deterministic mitigation of local trapping (a significant problem for many optimization algorithms). It also may lead to a new proof of convergence (a milestone achievement for any algorithm). Applying NEO theory likely requires collaboration between theorists in celestial mechanics and optimization. Hopefully these observations will stimulate that collaboration.

2 Methodology

CFO locates the global maxima (fitnesses) of a scalar-valued objective function $f(x_1, x_2, ..., x_N)$ with unknown topology (landscape) defined on an *N*-dimensional (*n*-D) decision space (DS). CFO [1-3] is a Nature-inspired metaheuristic like Particle Swarm

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Optimization and Ant Colony Optimization. But unlike PSO and ACO, it is deterministic instead of stochastic. CFO analogizes gravitational kinematics, thus embracing the metaphor of Newton's precise laws of gravity and motion. Under certain conditions small objects moving through space close to our planet (NEOs) can be gravitationally captured. Such encounters modify the NEO's orbit, at least for a while. In the absence of energy dissipation, orbital changes may persist for some time while the NEO and planet earth conservatively exchange energy. The NEO has been "trapped" in proximity to the earth, and it is this effect that the CFO metaphor embraces. Of course, CFO is an algorithm, a step-by-step procedure for processing numbers. It is not literally, nor is it intended to be, a precise model of how small masses move through space on paths bringing them close to a planet (indeed, the problem of calculating the motion of even three gravitating bodies remains unsolved). CFO thus is a conceptual approach to multidimensional search and optimization, a metaheuristic, drawing its inspiration from gravitational kinematics and, in a formal way, reflecting the equations underlying gravitational motion. But the similarity ends there. CFO also is similar in some ways to gradient-based optimization methods as discussed in [4]. Proofs of convergence for CFO and an extended version have been developed [5,6], and the algorithm has been implemented on a GPU using various topologies [7-9]. The algorithm has been successfully applied to a variety of problems, among them: training neural networks [10]; power grid reliability assessment [11]; drinking water distribution networks [12]; solving nonlinear circuits [13]; array synthesis [14,15]; microstrip patch antenna design [16]; multiband slotted bowtie design [17]; rectangular microstrip patch design [18]; microwave broadband absorber design [19]; antenna optimization generally [20]; notched ultra wideband E-shape antenna design [21]; and increasing impedance bandwidth [22,23].

The NEO-CFO connection is illustrated using two well-known benchmark functions: (1) the *n*-D step function in 2D (which can be visualized); and (2) the Griewank function in 30D. The step is defined as $f(x) = -\sum_{i=1}^{N_d}\left(\lfloor x_i - x_o^i + 0.5 \rfloor\right)^2$, $-100 \le x_i \le 100$ (here $N_d = 2$, $x_o^1 = 75$, $x_o^2 = 30$). This highly discontinuous function is unimodal with a maximum value of zero offset to the point $(75,30)$. Figs. 1(a) and (b) plot it over its domain and in the vicinity of the maximum.

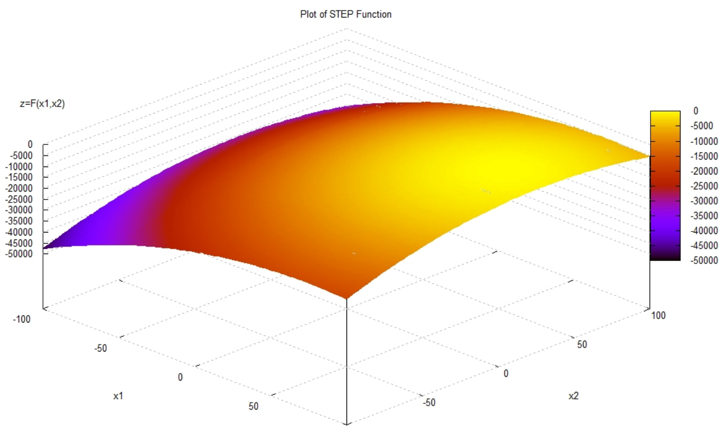

Fig. 1(a). 2D Step over its domain

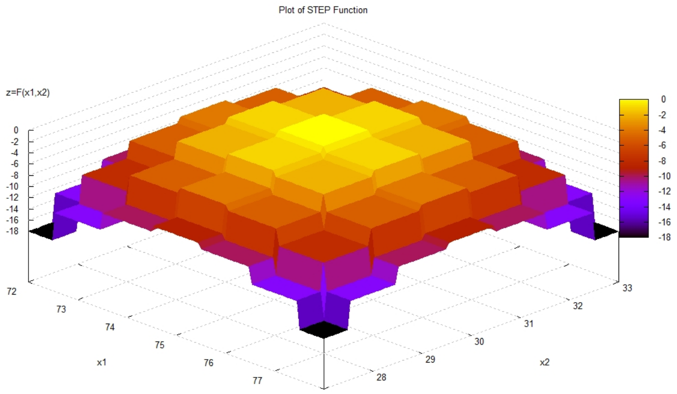

Fig. 1(b). 2D Step near the global maximum

The 30-dimensional modified Griewank function is defined as

$$f(\vec{x}) = -\frac{1}{4000}\sum_{i=1}^{30}(x_i - x_0)^2 + \prod_{i=1}^{30}\cos\left(\frac{x_i - x_0}{\sqrt{i}}\right) - 1\text{, where } -600 \le x_i \le 600\text{, } x_0 = 75.123.$$

The Griewank's global maximum value is zero at the offset point $x_i = 75.123,\ i = 1,..,30$. This function is extremely multimodal, and one of the most challenging benchmark functions because the number of local maxima increases exponentially with increasing decision space dimensionality [24]. In addition, offsetting the maximum from the origin ($x_0 = 0$) to a substantially distant point ($x_0 = 75.123$) makes it even more difficult to locate the global maximum (it appears that an offset is uncommon in the literature). The Griewank's complexity is illustrated by the 2-dimensional version plotted in Fig. 2 in a truncated region around the maximum.

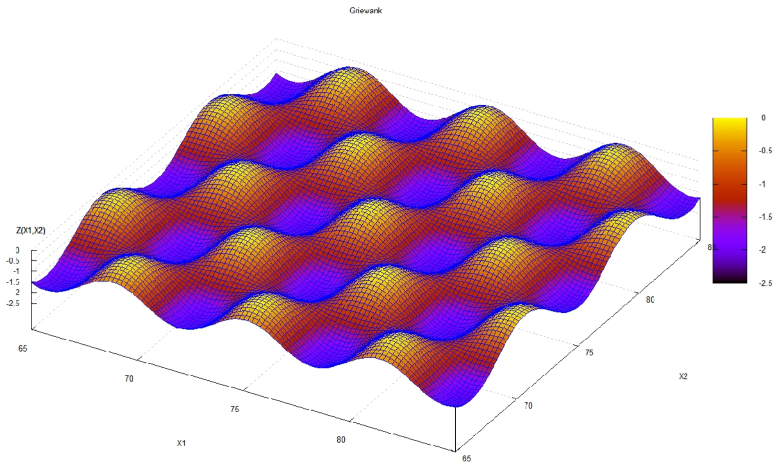

Fig. 2. 2D Griewank near the global maximum

CFO flies "probes" through DS over "time steps" (iterations). Their trajectories are computed from two *equations of motion* analogous to the equations of motion for masses moving through space under the influence of real gravity. CFO "mass" is created by defining a function of the objective function's fitness. The equations of motion for the probes' acceleration and position vectors are $\vec{a}_{j-1}^{\,p} = G \sum_{\substack{k=1 \\ k \neq p}}^{N_p} U(M_{j-1}^k - M_{j-1}^p) \cdot (M_{j-1}^k - M_{j-1}^p)^{\alpha} \times \frac{(\vec{R}_{j-1}^k - \vec{R}_{j-1}^p)}{\left\| \vec{R}_{j-1}^k - \vec{R}_{j-1}^p \right\|^{\beta}}$ and $\vec{R}_j^p = \vec{R}_{j-1}^p + \frac{1}{2}\vec{a}_{j-1}^{\,p}\,\Delta t^2,\ j \geq 1$, where $M_{j-1}^p = f(x_1^{p,j-1}, x_2^{p,j-1}, \ldots, x_N^{p,j-1})$, $\vec{R}_j^p = \sum_{k=1}^{N_d} x_k^{p,j}\hat{e}_k$, in which the $x_k^{p,j}$ are probe p 's coordinates at time step j, and $\hat{e}_k$ is the unit vector along the x_k axis. Note that $\vec{a}_{j-1}^{\,p} \equiv 0$ if $\vec{R}_{j-1}^k = \vec{R}_{j-1}^p$ to avoid an indeterminate form. Indices $1 \leq p \leq N_p$ and $0 \leq j \leq N_t$ are the probe and iteration numbers, N_p and N_t being the total numbers. $U(\cdot)$ is the Unit Step, $U(z) = \begin{Bmatrix} 1, & z \geq 0 \\ 0, & otherwise \end{Bmatrix}$, and $MASS_{CFO} = U(M_{j-1}^k - M_{j-1}^p) \cdot (M_{j-1}^k - M_{j-1}^p)^{\alpha}$. A measure of how well CFO's probes converge on a maximum is the average distance between the probe with the best fitness and all other probes at the j^{th} iteration normalized to the size of the decision space, that is, $D_{avg} = \frac{1}{L \cdot (N_p - 1)} \sum_{p=1}^{N_p} \sqrt{\sum_{i=1}^{N_d} \left[x_i^{p,j} - x_i^{p^*,j} \right]^2}$ where p^* is the number of the probe with the best fitness, $L = \sqrt{\sum_{i=1}^{N_d} (x_i^{\max} - x_i^{\min})^2}$ is the length of the decision space principal diagonal, and $x_i^{\min} \leq x_i \leq x_i^{\max}, i = 1,.., N_d$ defines DS (minimum/maximum values of each coordinate). The original CFO paper [1] included a velocity term that was set equal to zero as a matter of convenience because it simply was a additive constant in the case of rectilinear motion. It became clear upon further consideration that this term should not be included in equation at all because, in general, a probe's motion is not rectilinear. Instead it is curvilinear, so that the acceleration and velocity vectors are in different directions. In the case of circular motion, for example, the velocity vector is tangent to the trajectory circle while the acceleration is inwardly directed along the circle's radius, that is, perpendicular to the velocity. This limiting case illustrates why, in general, the velocity term appearing in real-world kinematic equations should not be included in metaphorical CFO-space because of how it effects the direction of a probe's acceleration.

3 Results and Discussion

Fig. 3(a) plots the 2D step function's probe trajectories for the probes with the best fitnesses, while Fig. 3(b) shows the individual probe trajectories ordered by probe number. These plots are visually chaotic, providing no hint whatsoever of the underlying mathematical regularity that forms the NEO-CFO nexus. That regularity appears in CFO's D_{avg} curve plotted in Fig. 4 (annotated with run parameters). D_{avg} exhibits four oscillatory plateaus connected by jumps. Although the oscillation may not be precisely repetitive, in many cases it is (for example, in this case starting at step 162 D_{avg} comprises the repeating sequence 0.6859416, 0.6917107, 0.6868708, 0.6952526, 0.6855014, 0.6956451, 0.6859433, 0.6917887, 0.6868326, 0.6859393, 0.6877515, 0.6939823, 0.6870971, 0.6956298, 0.6859431, 0.6866136, 0.6872625, 0.6940363, 0.6861267, 0.6953240, which presumably repeats indefinitely). Oscillation in D_{avg} appears to be a reliable signal of local trapping (determined empirically). In this case, CFO is trapped at a local maximum of -1 at $(75, 28.57142857)$. Trapping caused CFO to miss the global maximum, which often a problem with deterministic algorithms.

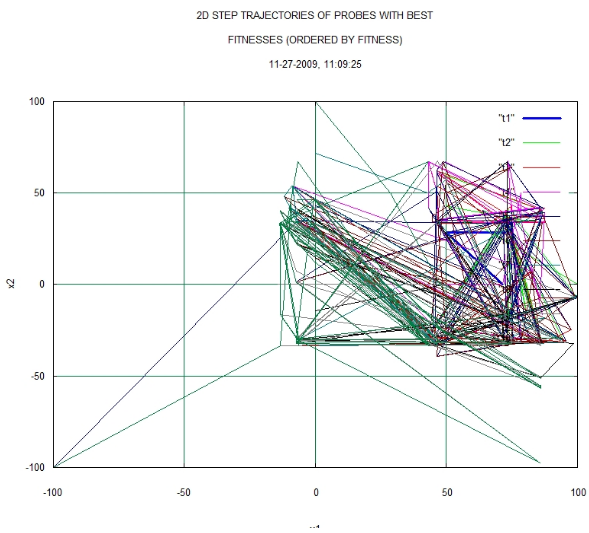

Fig. 3(a). Trajectories of probes with best fitnesses

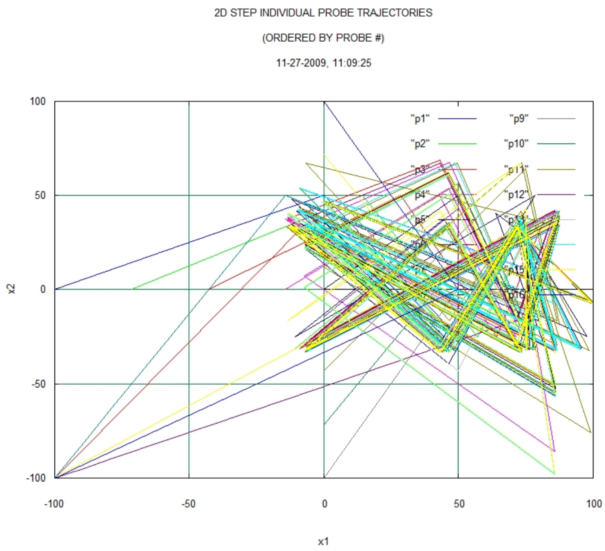

Fig. 3(b). CFO probe trajectories by probe number

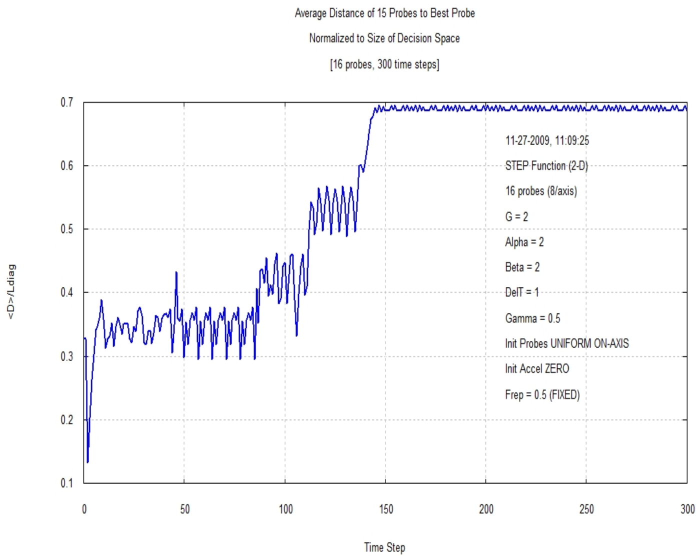

Fig. 4. 2D step D_{avg} vs. time step

For the 30D Griewank function, CFO returns a maximum fitness of -0.0030385 at step 151, and its D_{avg} curve is shown in Fig. 5. The oscillatory plateau-like regions connected by jumps seen in the step function's D_{avg} curve again are evident in this plot, and it is this characteristic that connects CFO to NEO's. CFO's D_{avg} curves under trapping are structurally similar to the ΔV

curve for a gravitationally trapped Near Earth Object. The similarity is obvious from Fig. 6, which plots asteroid Apophis' ΔV curve (reproduced from [25] with permission) computed by Professors Andrea Milani and Andrea Caruso using the theory of resonant returns [26] (private communication, Astronaut "Rusty" Schweickart). ΔV is the velocity change needed to avoid earth impact, and D_{avg} is a similar variable because it is proportional to velocity if Δt is constant. Another example of this effect using the Space Gravitational Optimization (SGO) benchmark function is described in detail in [27].

Apophis' ΔV curve in Fig. 6 contains two well-defined oscillatory plateaus connected by a jump and what appears to be the beginning of a third plateau, also connected by a jump, that is cut off by the vertical line marking earth impact in year 2036. The structural similarity to D_{avg} in Figs. 4 and 5 is striking. Both the D_{avg} and ΔV curves comprise oscillatory plateau-like regions connected by jumps, and it is difficult to imagine that their similarity is accidental. Rather, because the Apophis plot is based on real gravity trapping the asteroid in earth orbit, and D_{avg} is based on CFO's metaphorical gravity trapping a probe at a local (possibly global) maximum, it seems reasonable to speculate that the similarity actually may be inevitable.

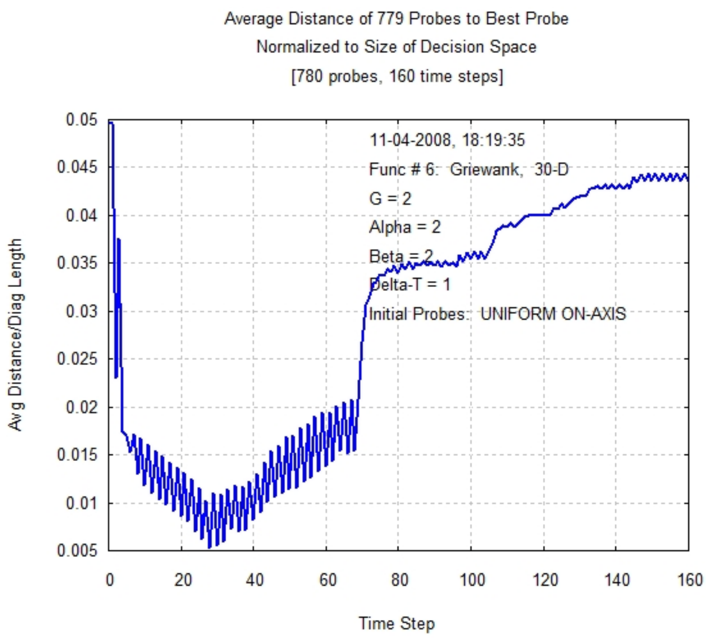

Fig. 5. 30D Griewank D_{avg} vs. time step

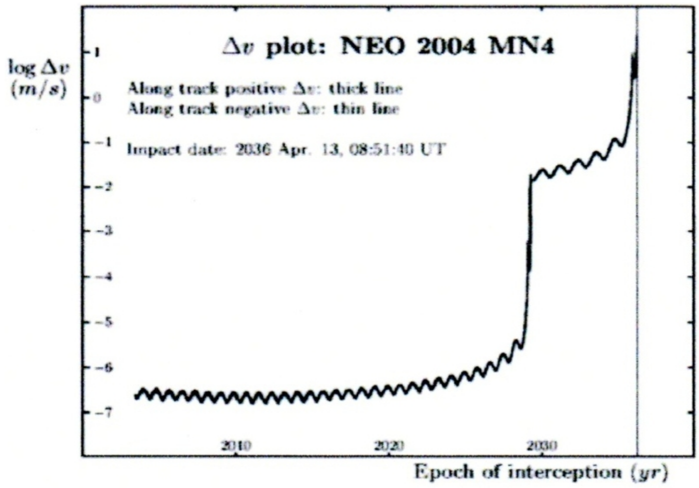

Fig. 6. Asteroid Apophis' ΔV plot

4 Conclusions

The unanswered question raised by this note is, Why are CFO's D_{avg} and an NEO's ΔV curves so similar in structure? Is it pure coincidence, which seems unlikely, or is it a consequence of CFO's gravitational metaphor reflecting gravitational kinematics as they affect actual physical objects? If the correct answer is the latter, then this observation is a compelling validation of the CFO gravitational metaphor, as well as the basis for speculating that NEO theory may hold the key to solving important problems in optimization. Hopefully researchers with appropriate skills and interests will find out if it does.

Competing Interests

Author has declared that no competing interests exist.

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