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Numerical Solution of Fractional Partial Differential-Algebraic Equation by Adomian Decomposition Method and Multivariate Pade Approximation

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Authors' contributions

This work was carried out in collaboration between all authors. Authors GDK and EÇ designed the study, performed the analysis and wrote the first draft of the manuscript. Authors GDK and EÇ obtained the ADM solution of Eq.1 and authors GDK, MY and EÇ got MPA of this solution. All authors read and approved the final manuscript.

Original Research Article

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ABSTRACT

In this study, Adomian Decomposition Method (ADM) and Multivariate Padé Approximation (MPA) are used to get solution of fractional partial differential algebraic equation (FPDAE).The solutions in the form of power series are obtained by using ADM first and then we get approximate solutions by means of MPA. While solving the equation, Caputo derivative is utilized. Methods are applied on a test problem. Results demonstrate that they are quite applicable.

Keywords: Fractional partial differential-algebraic equation; Adomian decomposition method; multivariate Padé Approximation.

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1. INTRODUCTION

Recently there has been much interest about differential equations of fractional order in various areas of physics and engineering [1,2,3,4]. To obtain the solution of this type of equations, approximation and numerical techniques must be used. Among them, (ADM) [5,6,7,8,9,10,11] is an effective one, as it solves linear/nonlinear differential equations. There exists various definitions and theorems of multivariate Padé approximations (MPAs) [12,13,14,15]. In the literature, unvariate and multivariate Padé approximation have been used to obtain approximate solutions of fractional order [16,17]. There are also numerical methods [18,19,20,21].

The objective of the present paper is to obtain approximate solution for the following type of problem by using MPA.

$$D_{*t}^{\alpha}u_i(x,t) = f_i\left(x,t,u_i,u_{i_x}\right), 0<\alpha\leq 1, \qquad i=1,2,\dots,n-1 \tag{1.a}$$

$$0 = \varphi(x,t,u_i) \tag{1.b}$$

subjecttotheinitialconditions

$$u_i(x,0)=a_i, \qquad i=1,2,\dots \tag{1.c}$$

2. DEFINITIONS

There are several definitions of fractional derivative of order $\alpha>0$ [22]. We give some basic definitions and properties of fractional calculus theory which are used in this paper.

Definition 2.1 A real function$f(x), x>0$is said to be in the space $C_\mu, \mu\epsilon R$ if there exists a real number $p>\mu$ such as $f(x)=x^p f_1(x)$, where $f_1(x)\epsilon C[0,\infty)$.

Definition 2.2 A function $f(x), x>0$ is said to be in the space C_μ^m iff $f^{(m)}\in C_\mu$, $m\in N$.

Definition2.3 The Riemann-Liouville fractional integral operator of order $\alpha>0$of a function$f\in C_\mu, \mu\geq -1$ is defined as

$$J^\alpha f(x)=\frac{1}{\Gamma(\alpha)}\int_0^x (x-t)^{\alpha-1}f(t)dt,\ x>0 \tag{2.1}$$

$$J^0 f(x)=f(x). \tag{2.2}$$

The properties of the operator J^α can be found in [23,24]: we mention only the following. For $f\in C_\mu,\ \mu\geq -1,\ \alpha,\beta\geq 0$ and $\gamma>-1$

$$\textbf{1.}J^\alpha J^\beta f(x)=J^{\alpha+\beta} \tag{2.3}$$

$$\textbf{2.}J^\alpha J^\beta f(x)=J^\beta J^\alpha f(x) \tag{2.4}$$

$$\textbf{3.}J^\alpha (x-a)^\gamma=\frac{\Gamma(\gamma+1)}{\Gamma(\alpha+\gamma+1)}.(x-a)^{\alpha+\gamma},\ \ \alpha>0,\ \gamma>-1,\ x>0 \tag{2.5}$$

Definition 2.4 The fractional derivative of $f(x)$ in the Caputo [25] is defined as

$$D_*^\alpha f(x) = J^{(m-\alpha)} D^m f(x) = \frac{1}{\Gamma(m-\alpha)} \int_0^x (x-t)^{m-\alpha-t} f^{(m)}(t) dt \quad (2.6)$$

for $m-1 < \alpha \leq m$, $m \in \mathbb{N}$, $x > 0$, $f \in C_{-1}^m$.

Also, here we need two of its basic properties.

Lemma 2.1 If $m-1 < m$, $m \in \mathbb{N}$, $f \in C_\mu^m$, $m \geq -1$, then

1. $D_*^\alpha J^\alpha f(x) = f(x)$ (2.7)

2. $J^\alpha D_*^\alpha f(x) = f(x) - \sum_{k=0}^{m-1} f^{(k)}(0^+) \frac{x^k}{k!},\ \ x > 0$ (2.8)

Let's revisit ADM for the differential equation

3. ADOMIAN DECOMPOSITION METHODS

Consider the general differential equation

$$Lu + Ru + Nu = g(x) \quad (3.1)$$

with the following initial condition

$$u(x,0) = f(x) \quad (3.2)$$

where L is linear operator which is assumed to be easily invertible, R is the remaining linear part, N is a nonlinear operator, and $g(x)$ is a known analytical function.

We can write Eq. (3.1) as

$$Lu = g(x) - Ru - Nu \quad (3.3)$$

Applying the inverse operator L^{-1} to Eq. (3.3), we get

$$u = f(x) - L^{-1}(Ru) - L^{-1}(Nu) \quad (3.4)$$

where $f(x) = L^{-1}(g(x)) + \varphi(x)$ and $\varphi(x)$is determined by initial value. The standard ADM [5,26,27,28,29] suggests that the solution $u(x,t)$ is decomposed by the infinite series of components

$$u = \sum_{n=0}^\infty u_n, \quad (3.5)$$

and the nonlinear term Nu is decomposed as follows:

$$Nu = \sum_{n=0}^\infty A_n, \quad (3.6)$$

where A_n are Adomian polynomials, defined by

$$A_n = \frac{1}{n!} \left[\frac{d^n}{d\lambda^n} N\left(\sum_{i=0}^\infty \lambda^i u_i\right) \right]_{\lambda=0}, n \geq 0 \quad (3.7)$$

Substituting (3.5) and (3.6) into both side of (3.4) we get

$$\sum_{n=0}^{\infty} u_n = f(x) - L^{-1}R\sum_{n=0}^{\infty} u_n - L^{-1}\sum_{n=0}^{\infty} A_n \tag{3.8}$$

The standard ADM defines u_n by the following recursive relationship

$$\begin{aligned} u_0 &= f(x) \\ u_{k+1} &= -L^{-1}[Ru_n + A_n] \end{aligned} \tag{3.9}$$

4. MULTIVARIATE PADÉ APPROXIMATION

Consider the bivariate function $f(x,y)$ with Taylor series development

$$f(x,y) = \sum_{i,j=0}^{\infty} c_{ij} x^i y^j \tag{4.1}$$

around the origin. For $f(x) = \sum_{i=0}^{\infty} c_i x^i$, let us examine a solution of unvariate Padé approximation.

$$p(x) = \begin{vmatrix} \sum_{i=0}^{m} c_i x^i & x\sum_{i=0}^{m-1} c_i x^i & \cdots & x^n \sum_{i=0}^{m-n} c_i x^i \\ c_{m+1} & c_m & \cdots & c_{m+1-n} \\ \vdots & \vdots & \ddots & \vdots \\ c_{m+n} & c_{m+n-1} & \cdots & c_m \end{vmatrix} \tag{4.2}$$

and

$$q(x) = \begin{vmatrix} 1 & x & \cdots & x^n \\ c_{m+1} & c_m & \cdots & c_{m+1-n} \\ \vdots & \vdots & \ddots & \vdots \\ c_{m+n} & c_{m+n-1} & \cdots & c_m \end{vmatrix} \tag{4.3}$$

Let us now multiply j throw of $p(x)$ and $q(x)$ by $x^{m+j-1} (j = 2, \dots, n+1)$ and then divide jth column of $p(x)$ and $q(x)$ by $x^{j-1} (j = 2, \dots, n+1)$ [13]. These results in a multiplication of numerator and denominator by x^{mn}. Thus we obtain

$$\frac{p(x)}{q(x)} = \frac{\begin{vmatrix} \sum_{i=0}^{m} c_i x^i & x\sum_{i=0}^{m-1} c_i x^i & \cdots & x^n \sum_{i=0}^{m-n} c_i x^i \\ c_{m+1} & c_m & \cdots & c_{m+1-n} \\ \vdots & \vdots & \ddots & \vdots \\ c_{m+n} & c_{m+n-1} & \cdots & c_m \end{vmatrix}}{\begin{vmatrix} 1 & x & \cdots & x^n \\ c_{m+1} & c_m & \cdots & c_{m+1-n} \\ \vdots & \vdots & \ddots & \vdots \\ c_{m+n} & c_{m+n-1} & \cdots & c_m \end{vmatrix}} \tag{4.4}$$

If $D = \det\ D_{m,n} \neq 0$.

This part of determinants can also be written down for $f(x,y)$ bivariate function. The sum $f(x) = \sum_{i=0}^{k} c_i x^i$ shall be replaced k. partial sum of the Taylor series development of $f(x,y)$ and the expression $c_k x^k$ by an expression that contains all the terms of degree k in $f(x,y)$. Here a bivariate term $c_{ij} x^i y^j$ is said to be of degree$i+j$.
If we define

$$p(x,y)=\begin{vmatrix}\sum_{i+j=0}^{m}c_{ij}x^iy^j & \sum_{i+j=0}^{m-1}c_{ij}x^iy^j & \cdots & \sum_{i+j=0}^{m-n}c_{ij}x^iy^j \\ \sum_{i+j=m+1}c_{ij}x^iy^j & \sum_{i+j=m}c_{ij}x^iy^j & \cdots & \sum_{i+j=m+1-n}c_{ij}x^iy^j \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{i+j=m+n}c_{ij}x^iy^j & \sum_{i+j=m+n-1}^{m}c_{ij}x^iy^j & \cdots & \sum_{i+j=m}^{m}c_{ij}x^iy^j\end{vmatrix} \quad (4.5)$$

and

$$q(x,y)=\begin{vmatrix}1 & 1 & \cdots & 1 \\ \sum_{i+j=m+1}c_{ij}x^iy^j & \sum_{i+j=m}c_{ij}x^iy^j & \cdots & \sum_{i+j=m+1-n}c_{ij}x^iy^j \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{i+j=m+n}c_{ij}x^iy^j & \sum_{i+j=m+n-1}^{m}c_{ij}x^iy^j & \cdots & \sum_{i+j=m}^{m}c_{ij}x^iy^j\end{vmatrix} \quad (4.6)$$

Here $p(x,y)$ and $q(x,y)$ are of the form

$$\begin{aligned}p(x,y)&=\sum_{i+j=mn}^{mn+m}a_{ij}x^iy^j\\ q(x,y)&=\sum_{i+j=mn}^{mn+n}b_{ij}x^iy^j\end{aligned} \quad (4.7)$$

$p(x,y)$ and $q(x,y)$ are called Padé equations [13]. Thus the multivariate Padé approximant of order (m,n) for $f(x,y)$ is defined as,

$$r_{m,n}(x,y)=\frac{p(x,y)}{q(x,y)} \quad (4.8)$$

5. NUMERICAL EXAMPLE

Let us consider the following fractional partial differential-algebraic equation

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&0\end{bmatrix}\begin{bmatrix}D_{*t}^{\alpha}u\\D_{*t}^{\alpha}v\\D_{*t}^{\alpha}w\end{bmatrix}+\begin{bmatrix}1&0&0\\0&0&0\\0&0&0\end{bmatrix}\begin{bmatrix}u_x\\v_x\\w_x\end{bmatrix}+\begin{bmatrix}1&0&1\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}u\\v\\w\end{bmatrix}=\begin{bmatrix}f_1\\f_2\\f_3\end{bmatrix} \quad (5.1)$$

with initial conditions

$$\begin{aligned}u(x,0)&=x\\ v(x,0)&=sinx\end{aligned} \quad (5.2)$$

where,

$$\begin{aligned}f_1&=2+x+t+xcos(x+t)\\ f_2&=\cos(x+t)+\sin(x+t)\\ f_3&=x\cos(x+t)\end{aligned} \quad (5.3)$$

and exact solutions are as follows

$$\begin{aligned}u(x,t)&=x+t\\ v(x,t)&=\sin(x+t)\\ w(x,t)&=xcos(x+t)\end{aligned} \quad (5.4)$$

Eq. (5.1) can be written as,

$$D_{*t}^{\alpha}u(x,t) = 2 + x + t - u_x(\mathrm{x},\mathrm{t}) - u(x,t)$$
$$D_{*t}^{\alpha}v(x,t) = \cos(x+t) + \sin(x+t) - v(x,t) \quad (5.5)$$

Using the inverse operator J^{α} and (5.2), we obtain

$$\begin{aligned} u(x,t) &= x + J^{\alpha}(2 + x + t) + J^{\alpha}(-u_x(\mathrm{x},\mathrm{t}) - u(x,t)) \\ v(x,t) &= \sin(x) + J^{\alpha}\big(\tilde{g}_1(x,t)\big) + J^{\alpha}(-v(x,t)) \end{aligned} \quad (5.6)$$

where $\tilde{g}_1(x,t)$ is Taylorseries of $g_1(x,t) = \sin(x+t) + \cos(x+t)$. Accordingly, the recursive relation is defined by,

$$\begin{aligned} u_0(x,t) &= u(x,0), \\ v_0(x,t) &= v(x,0), \\ u_1(x,t) &= J^{\alpha}(2+x+t) + J^{\alpha}\big(-u_{0_x} - u_0\big), \\ v_1(x,t) &= J^{\alpha}\big(\tilde{g}_1(x,t)\big) + J^{\alpha}(-v_0), \\ u_{k+1}(x,t) &= J^{\alpha}\big(-u_{k_x} - u_k\big), k \geq 1, \\ v_{k+1}(x,t) &= J^{\alpha}(-v_k),\ k \geq 1. \end{aligned} \quad (5.7)$$

$$u_0(x,t) = x$$
$$v_0(x,t) = sinx$$
$$u_1(x,t) = \frac{t^{\alpha}}{\Gamma(1+\alpha)} + \frac{t^{1+\alpha}}{\Gamma(2+\alpha)}$$
$$v_1(x,t) = \frac{t^{\alpha}}{\Gamma(1+\alpha)} + \frac{t^{\alpha}x}{\Gamma(1+\alpha)} - \frac{t^{\alpha}x^2}{2\Gamma(1+\alpha)} - \frac{t^{\alpha}x^3}{6\Gamma(1+\alpha)} + \frac{t^{1+\alpha}x}{\Gamma(2+\alpha)} - \frac{t^{1+\alpha}x}{\Gamma(2+\alpha)} - \frac{t^{1+\alpha}x^2}{2\Gamma(2+\alpha)} - \frac{t^{2+\alpha}}{\Gamma(3+\alpha)} - \frac{t^{2+\alpha}x}{\Gamma(3+\alpha)} - \frac{t^{3+\alpha}}{\Gamma(4+\alpha)} - \frac{t^{\alpha}\sin\ x}{\Gamma(1+\alpha)}$$
$$u_2(x,t) = -\frac{t^{2\alpha}}{\Gamma(1+2\alpha)} - \frac{t^{1+2\alpha}}{\Gamma(2+2\alpha)}$$
$$v_2(x,t) = -\frac{t^{2\alpha}}{\Gamma(1+2\alpha)} - \frac{t^{2\alpha}x}{\Gamma(1+2\alpha)} + \frac{t^{2\alpha}x^2}{2\Gamma(1+2\alpha)} + \frac{t^{2\alpha}x^3}{6\Gamma(1+2\alpha)} - \frac{t^{1+2\alpha}}{\Gamma(2+2\alpha)} + \frac{t^{1+2\alpha}x}{\Gamma(2+2\alpha)} + \frac{t^{1+2\alpha}x^2}{2\Gamma(2+2\alpha)} + \frac{t^{2+2\alpha}}{\Gamma(3+2\alpha)} + \frac{t^{2+2\alpha}x}{\Gamma(3+2\alpha)} + \frac{t^{3+2\alpha}}{\Gamma(4+2\alpha)} + \frac{t^{2\alpha}\sin\ x}{\Gamma(1+2\alpha)} \quad (5.8)$$
$$\vdots$$

and so on; in this manner, the rest of components of the decomposition series can be obtained.

The first four terms of the decomposition series are given by.

$$u(x,t) = \sum_{i=0}^{3} u_i(x,t) = x + \frac{t^{\alpha}}{\Gamma(1+\alpha)} + \frac{t^{1+\alpha}}{\Gamma(2+\alpha)} - \frac{t^{2\alpha}}{\Gamma(1+2\alpha)} - \frac{t^{1+2\alpha}}{\Gamma(2+2\alpha)} + \frac{t^{3\alpha}}{\Gamma(1+3\alpha)} + \frac{t^{1+3\alpha}}{\Gamma(2+3\alpha)} \quad (5.9)$$

$$v(x,t)=\sum_{i=0}^{3}v_i(x,t)$$
$$=\frac{t^{\alpha}}{\Gamma(1+\alpha)}+\frac{t^{\alpha}x}{\Gamma(1+\alpha)}-\frac{t^{\alpha}x^2}{2\Gamma(1+\alpha)}-\frac{t^{\alpha}x^3}{6\Gamma(1+\alpha)}+\frac{t^{1+\alpha}}{\Gamma(2+\alpha)}-\frac{t^{1+\alpha}x}{\Gamma(2+\alpha)}$$
$$-\frac{t^{1+\alpha}x^2}{2\Gamma(2+\alpha)}-\frac{t^{2+\alpha}}{\Gamma(3+\alpha)}-\frac{t^{2+\alpha}x}{\Gamma(3+\alpha)}-\frac{t^{3+\alpha}}{\Gamma(4+\alpha)}-\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}-\frac{t^{2\alpha}x}{\Gamma(1+2\alpha)}$$
$$+\frac{t^{2\alpha}x^2}{2\Gamma(1+2\alpha)}+\frac{t^{2\alpha}x^3}{6\Gamma(1+2\alpha)}-\frac{t^{1+2\alpha}}{\Gamma(2+2\alpha)}+\frac{t^{1+2\alpha}x}{\Gamma(2+2\alpha)}+\frac{t^{1+2\alpha}x^2}{2\Gamma(2+2\alpha)} \quad (5.10)$$
$$+\frac{t^{2+2\alpha}}{\Gamma(3+2\alpha)}+\frac{t^{2+2\alpha}x}{\Gamma(3+2\alpha)}+\frac{t^{3+2\alpha}}{\Gamma(4+2\alpha)}+\frac{t^{3\alpha}}{\Gamma(1+3\alpha)}+\frac{t^{3\alpha}x}{\Gamma(1+3\alpha)}-\frac{t^{3\alpha}x^2}{2\Gamma(1+3\alpha)}$$
$$-\frac{t^{3\alpha}x^3}{6\Gamma(1+3\alpha)}+\frac{t^{1+3\alpha}}{\Gamma(2+3\alpha)}-\frac{t^{1+3\alpha}x}{\Gamma(2+3\alpha)}-\frac{t^{1+3\alpha}x^2}{2\Gamma(2+3\alpha)}-\frac{t^{2+3\alpha}}{\Gamma(3+3\alpha)}$$
$$-\frac{t^{2+3\alpha}x}{\Gamma(3+3\alpha)}-\frac{t^{3+3\alpha}}{\Gamma(4+3\alpha)}+\sin x-\frac{t^{\alpha}\sin x}{\Gamma(1+\alpha)}+\frac{t^{2\alpha}\sin x}{\Gamma(1+2\alpha)}-\frac{t^{3\alpha}\sin x}{\Gamma(1+3\alpha)}$$

Firstly let us write $sinx \cong x-\frac{x^3}{6}+\frac{x^5}{120}$.

For $\alpha=1$, MPA of u the function can be calculated when $m=3$ and $n=1$ as follows:

$$u(x,t)=x+t+0.0446666667t^4 \quad (5.11)$$

$$[3,1]_{u(x,t)} = \frac{p_u}{q_u} \quad (5.12)$$

$$[3,1]_{u(x,t)} = \frac{\begin{vmatrix} x+t & x+t \\ 0.04166666667t^4 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 0.04166666667t^4 & 0 \end{vmatrix}} \quad (5.13)\ (5.14)$$

$$[3,1]_{u(x,t)} = x+t$$

and

$$v(x,t)=x+t-0.1666666667x^3+0.008333333333x^5-0.1666666667t^3$$
$$+0.04166666667t^4-0.001388888889t^6-0.008333333333t^5x$$
$$-0.5t^2x-0.5tx^2-0.02083333334t^4x^2-0.0083333333tx^5$$
$$+0.0041666666t^2x^5-0.0013888888t^3x^5 \quad (5.15)$$

Similarly, for $\alpha=1$, MPA of v function can be calculated when $m=8$ and $n=2$ as follows:

$$[8,2]_{v(x,t)}=\frac{p_v}{q_v} \quad (5.16)$$

$$[8,2]_{v(x,t)}=\frac{\begin{vmatrix} K & L & M \\ 0 & -0.0013888888t^3x^5 & 0.0041666666t^2x^5 \\ 0 & 0 & -0.0013888888t^3x^5 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 0 & -0.0013888888t^3x^5 & 0.0041666666t^2x^5 \\ 0 & 0 & -0.0013888888t^3x^5 \end{vmatrix}} \quad (5.17)$$

$$[8,2]_{v(x,t)} = x+t-0.1666666667x^3+\cdots-0.0013888888t^3x^5 \quad (5.18)$$

where

$$K = x + t - 0.1666666667x^3 + 0.008333333333x^5 - 0.1666666667t^3 + 0.04166666667t^4 - 0.001388888889t^6 - 0.008333333333t^5x - 0.5t^2x - 0.5tx^2 - 0.02083333334t^4x^2 - 0.0083333333tx^5 + 0.0041666666t^2x^5 - 0.0013888888t^3x^5$$

$$L = x + t - 0.1666666667x^3 + 0.008333333333x^5 - 0.1666666667t^3 + 0.04166666667t^4 - 0.001388888889t^6 - 0.008333333333t^5x - 0.5t^2x - 0.5tx^2 - 0.02083333334t^4x^2 - 0.0083333333tx^5 + 0.0041666666t^2x^5$$

$$M = x + t - 0.1666666667x^3 + 0.008333333333x^5 - 0.1666666667t^3 + 0.04166666667t^4 - 0.001388888889t^6 - 0.008333333333t^5x - 0.5t^2x - 0.5tx^2 - 0.02083333334t^4x^2 - 0.0083333333tx^5$$

Byusing ADM, MPA solutions obtained as $u_{[11,2]}, v_{[21,2]}$ for $\alpha = 0.75$; $u_{[3,2]}, v_{[9,2]}$ for $\alpha = 0.5$.

Table 1. Numerical results of $u(x,t)$ ($t = 0.01$)

x	$\alpha = 0.5$		$\alpha = 0.75$	
	u_{ADM}	u_{MPA}	u_{ADM}	u_{MPA}
0	0.1042954313	0.1042953034	0.0338614405	0.0338614024
0.1	0.2042954313	0.2042953034	0.1338614406	0.1338614024
0.2	0.3042954313	0.3042953034	0.2338614406	0.2338614024
0.3	0.4042954313	0.4042953034	0.3338614406	0.3338614024
0.4	0.5042954313	0.5042953034	0.4338614406	0.4338614024
0.5	0.6042954313	0.6042953034	0.5338614406	0.5338614024
0.6	0.7042954313	0.7042953034	0.6338614406	0.6338614024
0.7	0.8042954313	0.8042953034	0.7338614406	0.7338614024
0.8	0.9042954313	0.9042953034	0.8338614406	0.8338614024
0.9	1.0042955303	1.0042953034	0.9338614406	0.9338614024
1.0	0.1042954313	1.1042953034	1.0338614410	1.0338614020

x	$\alpha = 1$				
	u_{ADM}	u_{MPA}	$u_{TÇ}$	$\lvert u_{TÇ} - u_{ADM}\rvert$	$\lvert u_{TÇ} - u_{MPA}\rvert$
0	0.0100000042	0.01	0.01	$0.42\ 10^{-9}$	0
0.1	0.1100000004	0.11	0.11	$0.4\ 10^{-9}$	0
0.2	0.2100000004	0.21	0.21	$0.4\ 10^{-9}$	0
0.3	0.3100000004	0.31	0.31	$0.4\ 10^{-9}$	0
0.4	0.4100000004	0.41	0.41	$0.4\ 10^{-9}$	0
0.5	0.5100000004	0.51	0.51	$0.4\ 10^{-9}$	0
0.6	0.6100000004	0.61	0.61	$0.4\ 10^{-9}$	0
0.7	0.7100000004	0.71	0.71	$0.4\ 10^{-9}$	0
0.8	0.8100000004	0.81	0.81	$0.4\ 10^{-9}$	0
0.9	0.9100000004	0.91	0.91	$0.4\ 10^{-9}$	0
1.0	1.0100000000	1.01	1.01	0	0

Table 2. Numerical results of $v(x,t)$ ($t=0.01$)

X	$\alpha = 0.5$		$\alpha = 0.75$	
	v_{ADM}	v_{MPA}	v_{ADM}	v_{MPA}
0	0.1042925722	0.1042925721	0.03386073224	0.03386073224
0.1	0.2035336918	0.2035336916	0.1335054038	0.1335054038
0.2	0.3007340959	0.3007340981	0.2318138743	0.2318138768
0.3	0.3949049573	0.3949049962	0.3277981885	0.3277982304
0.4	0.4850752287	0.4850755195	0.4204895568	0.4204898704
0.5	0.5703003089	0.5703016935	0.5089476997	0.5089491924
0.6	0.6496704464	0.6496754005	0.5922699051	0.5922752457
0.7	0.7223187943	0.7223333425	0.6695997128	0.6696153958
0.8	0.7874290368	0.7874660070	0.7401351345	0.7401749884
0.9	0.8442425101	0.8443266297	0.8031363312	0.8032270123
1.0	0.8920647479	0.8922401590	0.8579326691	0.8581217628

x	$\boldsymbol{\alpha = 1}$				
	$\boldsymbol{v_{ADM}}$	$\boldsymbol{v_{MPA}}$	$\boldsymbol{v_{TÇ}}$	$\boldsymbol{\lvert v_{TÇ} - v_{ADM} \rvert}$	$\boldsymbol{\lvert v_{TÇ} - v_{MPA} \rvert}$
0	0.009999833750	0.009999833750	0.00999983334	$0.416\ 10^{-9}$	$0.416\ 10^{-9}$
0.1	0.1097782495	0.1097782496	0.1097783008	$0.513\ 10^{-7}$	$0.512\ 10^{-7}$
0.2	0.2084591380	0.2084591406	0.2084598998	$0.7618\ 10^{-6}$	$0.7592\ 10^{-6}$
0.3	0.3050548392	0.3050548823	0.3050586364	$0.37972\ 10^{-5}$	$0.37541\ 10^{-5}$
0.4	0.3985973302	0.3985976514	0.3986093280	0.0000119978	0.0000116766
0.5	0.4881477963	0.4881493259	0.4881772469	0.0000294506	0.0000279210
0.6	0.5728059141	0.5728113860	0.5728674601	0.0000615460	0.0000560741
0.7	0.6517187461	0.6517348143	0.6518337710	0.0001150249	0.0000989567
0.8	0.7240891641	0.7241299964	0.7242871744	0.0001980103	0.0001571780
0.9	0.7891837143	0.7892766213	0.7895037397	0.0003200254	0.0002271184
1.0	0.8463398472	0.8465335824	0.8468318446	0.0004919974	0.0002982622

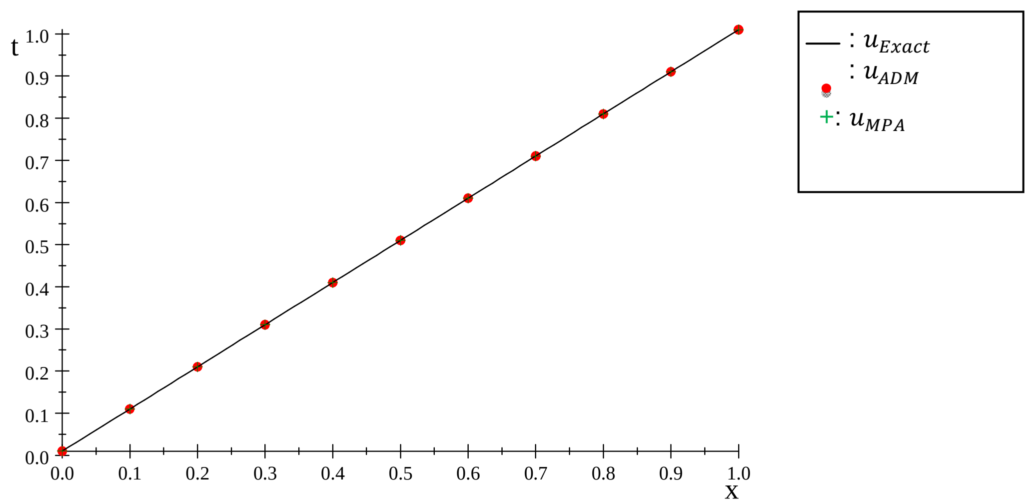

Fig. 1. Graphics of exact solution of u, ADM solution of u and MPA solution of u

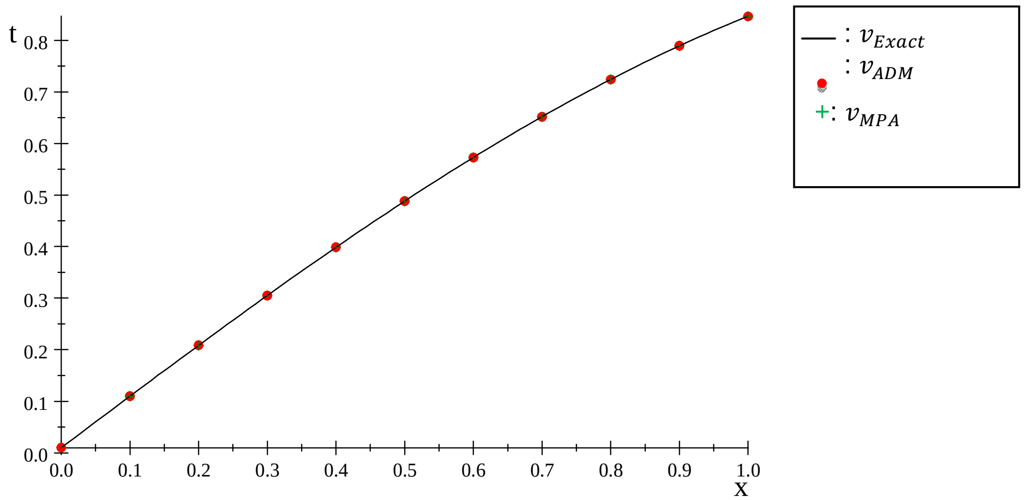

Fig. 2. Graphics of exact solution of v, ADM solution of v and MPA solution of v.

6. CONCLUSIONS

The process is illustrated by a numerical example. It is shown that ADM and MPA are very effective and suitable. When the tables and the figures above are analysed we reach this results: for $\alpha = 1$, solutions of ADM and MPA are in agreement with the exact solution and for the different values of α,they are in agreement with each other. The results indicate that MPA is convenient for solving fractional partial differential algebraic equations. On the other hand the results are quite reliable. Therefore, this method can be applied to many fractional partial differential algebraic equations.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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