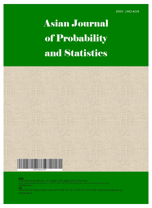

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Detection of Non-Normality in Data Sets and Comparison between Different Normality Tests

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Abstract

The paper provides five tests of data normality at different sample sizes. The tests are the Shapiro-Wilk (SW) test, Anderson-Darling (AD) test, Kolmogorov-Smirnov (KS) test, Ryan-Joiner (RJ) test, and Jarque-Bera (JB) test. These tests were used to test for normality for two secondary data sets with sample size (155) for large and (40) for small; and then test the simulated scenario with standard normal "N(0,1)" data sets; where the large samples of sizes (150, 140, 130, 130, 110 and 100) and small samples of sizes (40. 35, 30, 25, 20, 15 and 10) are considered at two levels of significance (5% and 10%). However, the aim of this paper is to detect and compare the performance of the different normality tests considered. The normality test results shows Kolmogorov-Smirnov (KS) test is a most powerful test than other tests since it detect the simulated large sample data sets do not follow a normal distribution at 5%, while for small sample sizes at 5% level of significance; the results showed the Jarque-Bera (JB) test is a most powerful test than other tests since it detects that the simulated small sample data do not follow a normal distribution at 5%. This paper recommended JB test for normality test when the sample size is small and KS test when the sample size is large at 5% level of significance.

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1 Introduction

This study compared five tests of data normality check, where these tests are performed to examine whether or not the observations considered follow a normal distribution. When a variable is normally distributed, then parametric statistics are used based on this assumption. More often large sample size is required to detect departures from normality. Only extreme types of non-normality can be detected with samples less than fifty observations because generally normality test has small statistical power (probability of detecting non-normal data) except the sample sizes are at least over 100. Statistical errors are common in scientific literature, and about 50% of the published articles have at least one error [1]. Many of the statistical procedures including correlation, regression, t-tests, and analysis of variance, namely parametric tests, are based on the assumption that the data follows a normal distribution or a Gaussian distribution; that is, it is assumed that the populations from which the samples are taken are normally distributed [2,3]. The assumption of normality is especially critical when constructing reference intervals for variables [4]. Normality and other assumptions should be taken seriously, for when these assumptions do not hold, it is impossible to draw accurate and reliable conclusions about reality [5].

With large enough sample sizes greater than thirty, the violation of the normality assumption should not cause major problems [6]; this implies that parametric procedures can be used, even when the data are not normally distributed [7]. If we have samples consisting of hundreds of observations, the distribution of the data can be ignored [3]. It is important to ascertain whether data show a serious deviation from normality [7].

The aim of this paperwork is to compare the performance of some of the methods for detecting normality. The objectives are: (1) To test for normality using five different statistical tests. (2) Ascertain the tests that were able to detect non-normality at different levels of Significance [5% and 10%] for both large and small samples.

2 Methods

This section includes the definitions and some terms associated with the analysis. The methods adopted in this study are some tests for normality checking. This section provides details of the five normality tests used in the study.

2.1 Shapiro-Wilk (W or SW Test)

The basic approach used in the **Shapiro-Wilk** (SW) test for normality is as follows:

1) Rearrange the data in ascending order so that $x_1 \leq \ldots \leq x_n$.
2) Calculate *SS* as follows:

$$SS = \sum_{i=1}^{n} \left(x_i - \bar{x} \right)^2 \tag{1}$$

3) If n is even, let $m = n/2$, while if n is odd let $m = (n–1)/2$
4) Calculate b as follows, taking the a_i weights from Table 1 (based on the value of n) in the Shapiro-Wilk Tables. Note that if n is odd, the median data value is not used in the calculation of b.

$$b = \sum_{i=1}^{m} a_i (x_{n+1-i} - x_i) \tag{2}$$

5) Calculate the test statistic

$$W = b^2 / SS \tag{3}$$

6) Find the value in Table 2 of the Shapiro-Wilk Tables (for a given value of n) that is closest to W, interpolating if necessary. This is the p-value for the test.

2.2 Jarque – Bera (JB) test

It is a better goodness-of-fit test that is used to test whether sample data has the skewness and kurtosis matching a normal distribution; which its statistic has a chi-square distribution with two degrees of freedom $\chi^2(2)$.

Hypothesis:

H_0: the data is normally distributed

against

H_1: the data is not normally distributed

where

Skewness: $$\gamma_1 = \frac{\sum_{i=1}^{n} \left(d_i - \bar{d}\right)^3}{ns^3} \tag{4}$$

Kurtosis: $$\gamma_2 = \frac{\sum_{j=1}^{n} \left(d_j - \bar{d}\right)^4}{ns^4} \tag{5}$$

Then,

$$JB = n\left(\frac{\gamma_1^{\,2}}{6} + \frac{\gamma_2^{\,2}}{24}\right) \approx \chi^2(2) \tag{6}$$

where

d is the difference in each observation.
n is the sample size.
s is the standard deviation.
γ_1 is the skewness.
γ_2 is the kurtosis.

Decision rule; we reject H_0, if JB > $\chi^2(2)$, otherwise accept H_0

Equation (6) was used to obtain the test statistic. We used the Jarque- Bera (JB) Test to test our data for normality. In Statistics, the JB Test is goodness of–fit test of whether sample data have the skewness and kurtosis matching a normal distribution.

Hypothesis:

H_0: Data follows a standard normal distribution; against
H_1: Data does not follow a standard normal distribution.

Decision rule; we reject H_0, if JB > $\chi^2(2)$, otherwise accept H_0

2.3 Kolmogorov-Smirnov test for normality (KS Test)

Definition 1: Let $x_1,\ldots,x_n$ be an ordered sample with $x_1 \le \ldots \le x_n$ and define $S_n(x)$ as follows:

$$S_n(x) = \begin{cases} 0, & x < x_1 \\ k/n, & x_k \le x \le x_{k+1} \\ 1, & x \ge x_n \end{cases} \tag{7}$$

Now suppose that the sample comes from a population with cumulative distribution function $F(x)$ and define D_n as follows:

$$D_n = \max_x \left| F(x) - S_n(x) \right| \tag{8}$$

Observation: It can be shown that D_n doesn't depend on F. Since S_n(x) depends on the sample chosen, D_n is a random variable. Our objective is to use D_n as a way of estimating $F(x)$.

The distribution of D_n will be calculated using statistical software, but now the important aspect of this distribution are the critical values.

If $D_{n,\alpha}$ is the critical value from the table, then $P(D_n \le D_{n,\alpha}) = 1 - \alpha$. D_n can be used to test the hypothesis that a random sample came from a population with a specific distribution function $F(x)$. If

$$\max_x \left| F(x) - S_n(x) \right| \le D_{n,\alpha} \tag{9}$$

Then the sample data is a good fit with $F(x)$. Also from the definition of D_n given above, it follows that

$$\begin{aligned} 1-\alpha = P\left(D_n \le D_{n.\alpha}\right) &= P\left(\max_x \left| F(x) - S_n(x) \right| \le D_{n,\alpha}\right) \\ &= P\left(S_n(x) - D_{n,\alpha} \le F(x) \le S_n(x) - D_{n,\alpha} \;\; for\ all\ x\right) \\ &= P\left(\left| F(x) - S_n(x) \right| \le D_{n,\alpha} \;\; for\ all\ x\right) \end{aligned} \tag{10}$$

Thus $S_n(x) \pm D_{n,\alpha}$ provides a confidence interval for F(x).

This test for normality is based on the maximum difference between the observed distribution and expected cumulative-normal distribution. Since it uses the sample mean and standard deviation to calculate the expected normal distribution, the Lilliefors' adjustment is used. The smaller the maximum difference the more likely that the distribution is normal.

This test has been shown to be less powerful than the other tests in most situations. It is included because of its historical popularity.

2.4 Anderson Darling (AD) test

Measures the area between the fitted line (based on chosen distribution) and the nonparametric step function (based on the plot points). The statistic is a squared distance that is weighted more heavily in the tails of the distribution. Smaller Anderson-Darling values indicate that the distribution fits the data better.

The Anderson-Darling normality test is defined as:

H_0: The data follow a normal distribution; against
H_1: The data do not follow a normal distribution

Test Statistic: The Anderson-Darling test statistic is defined as

$$(AD)^2 = N - \left(\frac{1}{N}\right)\sum_{i=1}^{N}(2i-1)\Big(\ln F(Y_i) + \ln\big(1 - F(Y_{N+1-i})\big)\Big) \tag{11}$$

where:

F is the cumulative distribution function of the normal distribution Y_i are the ordered observations and N is the sample size.

2.5 Ryan-Joiner (RJ) test

The Ryan-Joiner test provides a correlation coefficient of the ordered observations (Y_i), which indicates the correlation between your data and the normal scores of your data. If the correlation coefficient is close to 1, the data fall close to the normal probability plot. If it falls below the appropriate critical value, we will reject the null hypothesis of normality.

The correlation coefficient is calculated as:

$$RJ = \frac{\sum_{i=1}^{N} Y_i b_i}{\sqrt{S^2 (N-1)\sum_{i=1}^{N} b_i}} \tag{12}$$

where:

Y_i are ordered observations
b_i = normal scores of your ordered data
S^2 = sample variance.

In this work, the normality tests were applied to a large sample (n=155) with/without outliers and small sample (n=40) with/without outliers. First, identify the outliers, then went ahead to treat the outliers using the mean imputation and range test technique; then applied the normality tests again. In addition, we also transformed the data sets to standard normal and went ahead to confirm normality with these tests [8,9,10,11].

Next, comparison between the different normality tests was done of the transformed data sets (or differencing data set), the data set with Outliers (where detection of Outliers on the data set was done using Range test) and the data set without Outliers (treated data set was obtained using imputation technique called mean imputation method.

Differencing (Diff): refers to the transformation of time series data in order to achieve stationarity; It eliminates trend and seasonality which stabilizes the mean of the time series data. Scientifically, first-order differencing is expressed as $Y_t' = \nabla Y_t = y_t - y_{t-1}$. A stationary time series does not depend on time. Intermittently, second-order differencing is expressed as $Y_t'' = \nabla^2 Y_t = (1-B)^2 Y_t = Y_t - 2Y_{t-1} + Y_{t-2}$ and removes quadratic trends [12]. This process loses one a datum at each time is its disadvantage. Much natural time series are non–stationary. Box and Jenkins [13] proposed that differencing up to an appropriate order renders the data stationary for a non–stationary time series (X_t).

I) Outliers Detection: Outliers is detected with the use of range test in the series. Range Test: compute the overall mean and standard deviation of the data set. Then, subtract the mean from each observation values and divide by standard deviation, that is

$$R_T = \frac{x_i - \bar{x}}{\hat{\sigma}} \tag{13}$$

where: x – Extreme values, $\bar{X}$ - Overall Mean and $\hat{\sigma}$ - Overall standard deviation.

An extreme value (x) is an outlier if

$$\frac{abs(x - \bar{X})}{\hat{\sigma}} > 3 \tag{14}$$

II) Outliers Treatment (Mean Imputation Method):

The detected outliers were replace using the Mean imputation technique. This technique suggested that the outlier values are replaced with the mean data set (or Overall Mean).

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \tag{15}$$

Furthermore, a standard normal form with additive errors term of the data sets was obtained, then the different normality tests considered were done and compared. In addition, to compare these tests base on larges and sample sizes. We considered a simulation scenario for normality “N(0,1)” where sample sizes are 150, 140, 130, 130, 110 and 100 for large samples scenario; then the sample sizes are 40. 35, 30, 25, 20, 15 and 10 for small samples scenario. We used the normal data sets simulated to check how many times the hypothesis is rejected in each case.

3 Illustrations and Results

The common null hypothesis for these tests is H_0: data follow a normal distribution. If the p-value of the test is less than the α level used, reject H_0. In this section, two data sets were obtained to compare the performance of the five normality tests considered in the paper [Large sample (150) and a small sample (40)]. The first data set is with outliers (which was detected by range test and treated by mean imputation technique) and the second data set is without outliers.

3.1 Secondary data sets, illustrations and results

Illustration 1: Descriptive statistics and Normality tests for first data

Descriptive Statistics:

```
                   Total
Variable           Count    N  N*       Mean   StDev    Skewness  Kurtosis
Diff(NPA) with       155  155  0       -23.5    3236     -0.04      3.16
Diff(NPA) without    155  155  0       -70.4    2529     -0.02     -0.21
Diff(NPA) without    155  155  0  0.0000191  0.9999     -0.02     -0.21
Normal(0,1)          155  155  0      0.1163  0.8480     -0.02     -0.20
Additive             155  155  0       0.116   1.371     -0.04     -0.07
```

Table 3.1. Normality test of monthly revenue generated data of the Nigeria Ports Authority (NPA), Rivers Ports. Nigeria (2002 to 2014)

Normality test	Diff(NPA) with outlier	Decision at 5% and 10%	Diff(NPA) without outlier (after applied range test)	Diff(NPA) without outlier N(0,1)	Diff(NPA) with additive error	Decision at 5% and 10%
SW Test	**0.955 (0.000**)**	**Both rejected**	0.995 (0.879)	0.995 (0.879)	0.996 (0.943)	Accepted at both levels
AD Test	**1.140 (0.005**)**	**Both rejected**	0.263 (0.696)	0.263 (0.696)	0.129 (0.983)	Accepted at both levels
KS Test	**0.066 (0.095*)**	**Only rejected at 10%**	0.043 (>0.150)	0.043 (>0.150)	0.032 (0.200)	Accepted at both levels
RJ Test	**0.974 (<0.010**)**	**Both rejected**	0.998 (>0.100)	0.998 (>0.100)	0.998 (>0.100)	Accepted at both levels
JB Test	**64.532****	**Both rejected**	0.295	0.268	0.073	Accepted at both levels

*The p-values are in parenthesis. If p-value of the test is less than the α level (**=sig. at 5% and *= sig. at 10%), reject H_0 (or reject Normality) for SW, AD, KS, and RJ; while JB calculated greater than the $\chi^2(2)$ α level [5% (5.99) and 10% (4.61)], reject H_0*

Result: The normality test results in Table 3.1 show that four of the tests rejected H_0 for both levels of significance (5% and 10%) except the Kolmogorov-Smirnov Test (KS Test); which fail reject at 5% [Diff(NPA) with Outlier]. The data sets of Diff(NPA) without Outlier, Diff(NPA) without Outlier N(0,1) and Diff(NPA) with Additive error show that the different normality tests considered do not reject H_0 for both levels of significance (5% and 10%).

Illustration 2: Descriptive statistics and Normality tests for the second data set

Descriptive Statistics:

```
                    Total
Variable            Count     Mean  StDev   Variance  Skewness  Kurtosis
AMI                    40    17296   9615   92438639      0.86      0.71
Diff(AMI)              40     -429  12296  151198779     -0.53      0.82
Diff(AMI) N(0,1)       40   -0.247   6245   38999303     -0.53      0.82
```

Table 3.2. Normality test of Average Monthly Income (AMI) of respondents (naira) whose household utilized primary health care services two or more times in the last month [14]

Normality test	Average Monthly Income (AMI)	Decision at 5% and 10%	Diff(AMI)	Diff(AMI) N(0,1)	Decision at 5% and 10%
SW Test	**0.934 (0.025**)**	**Both rejected**	0.982 (0.782)	0.982 (0.782)	Accepted at both levels
AD Test	**0.840 (0.028**)**	**Both rejected**	0.202 (0.869)	0.202 (0.869)	Accepted at both levels
KS Test	**0.165 (<0.010**)**	**Both rejected**	0.065 (>0.150)	0.065 (>0.150)	Accepted at both levels
RJ Test	0.977 (>0.100)	Accepted at both levels	0.998 (<0.100)	0.998 (<0.100)	Accepted at both levels
JB Test	**5.77***	**Only rejected at 10%**	2.99	2.99	Accepted at both levels

*The p-values are in parenthesis. If p-value of the test is less than the α level (**=sig. at 5% and *= sig. at 10%), reject H_0 (or reject Normality) for SW, AD, KS, and RJ; while JB calculated greater than the $\chi^2(2)$ α level [5% (5.99) and 10% (4.61)], reject H_0*

Result: The normality test results in Table 3.2 shows that three of the tests rejected H_0 for both levels of significance (5% and 10%) except Jarque–Bera (JB) test which fails reject at 5% and Ryan-Joiner tests which fail to reject at both levels 5% and 10%) for the actual average monthly income (AMI) data. The Diff(AMI) data sets and Diff(AMI) standard normal form of N(0,1) shows that the different normality tests considered rejected H_0 for both levels of significance (5% and 10%).

3.2 Primary data sets, illustrations and results

Similarly, this section used two simulated scenario with standard normal "N(0,1)" data sets; where the large samples of sizes (150, 140, 130, 130, 110 and 100) and small samples of sizes (40. 35, 30, 25, 20, 15 and 10) are considered.

Illustration 3: Descriptive statistics and Normality tests for the first simulated data set

Descriptive Statistics: N=150, N=140, N=130, N=120, N=110, N=100

```
          Total
Variable  Count     Mean   StDev  Skewness  Kurtosis
N=150       150   0.0636  0.9991     -0.29     -0.33
N=140       140  -0.0547  0.9703      0.24     -0.24
N=130       130   0.0230  0.9911      0.03      0.00
N=120       120  -0.0001  0.8695      0.21     -0.04
N=110       110   0.1042  0.9279     -0.25      0.18
N=100       100  -0.0466  0.8632     -0.21      0.02
```

Table 3.3. Normality test of simulated large sample sizes

Normality test	Large samples of sizes					
	N=150	N=140	N=130	N=120	N=100	N=100
SW Test	0.985	0.992	0.986	0.982	0.992	0.991
	(0.336)	(0.792)	(0.371)	(0.180)	(0.785)	(0.755)
AD Test	0.621	0.401	0.211	0.709	0.270	0.181
	(0.104)	(0.356)	(0.855)	(0.063*)	(0.672)	(0.912)
KS Test	0.082	0.051	0.051	0.099	0.053	0.040
	(0.097*)	(0.200)	(0.200)	(0.017**)	(0.200)	(0.200)
RJ Test	0.994	0.996	0.996	0.993	0.995	0.996
	(>0.100)	(>0.100)	(>0.100)	(>0.100)	(>0.100)	(>0.100)
JB Test	2.783	1.680	0.020	0.890	1.294	0.737

*The p-values are in parenthesis. If p-value of the test is less than the α level (**=sig. at 5% and *= sig. at 10%), reject H_0; while JB calculated greater than the $\chi^2(2)$ α level [5% (5.99) and 10% (4.61)], reject H_0*

Result: From Table 3.3, the AD test rejected H_o at 10% for sample size 120, while the KS test rejected H_o at 5% for sample size 120 and rejected H_o at 10% for sample size 150. However, the other tests (SW, RJ and JB) do not reject H_o at both significance levels.

Illustration 4: Descriptive statistics and Normality tests for the second simulated data set

Descriptive Statistics: n=40, n=35, n=30, n=25, n=20, n=15, n=10

```
          Total
Variable  Count    Mean  StDev  Variance  Skewness  Kurtosis
n=40         40   0.184  0.884     0.782     -0.04     -1.25
n=35         35   0.209  0.843     0.710      0.61     -0.29
n=30         30  -0.125  0.825     0.681      0.69      0.58
n=25         25   0.165  1.101     1.212     -0.05      0.85
n=20         20   0.110  0.780     0.609      0.86      1.18
n=15         15   0.455  1.153     1.331     -0.08      0.07
n=10         10  -0.327  0.798     0.637      1.01      0.45
```

Table 3.4. Normality test of simulated small sample sizes

Normality test	Small samples of sizes						
	n=40	n=35	n=30	n=25	n=20	n=15	n=10
SW Test	0.957	0.853	0.960	0.968	0.940	0.957	0.957
	(0.751)	(0.063*)	(0.784)	(0.868)	(0.550)	(0.753)	(0.747)
AD Test	0.725	0.437	0.340	0.281	0.311	0.178	0.204
	(0.054*)	(0.280)	(0.474)	(0.611)	(0.523)	(0.902)	(0.824)
KS Test	0.202	0.193	0.131	0.119	0.156	0.130	0.120
	(0.200)	(0.200)	(0.200)	(0.200)	(0.200)	(0.200)	(0.200)
RJ Test	0.971	0.969	0.984	0.986	0.978	0.985	0.978
	(0.050*)	(0.059*)	(>0.100)	(>0.100)	(>0.100)	(>0.100)	(>0.100)
JB Test	5.297*	8.743**	1.418	0.133	0.594	0.363	0.753

*The p-values are in parenthesis. If p-value of the test is less than the α level (**=sig. at 5% and *= sig. at 10%), reject H_0; while JB calculated greater than the $\chi^2(2)$ α level [5% (5.99) and 10% (4.61)], reject H_0*

Result: From Table 3.4, SW test rejected H_o at 10% for sample size 35, AD test rejected H_o at 10% for sample size 40, RJ test rejected H_o at 10% for sample sizes 40 and 35, while JB test rejected H_o at 10% for sample size 40 and at 5% for sample size 35. KS test rejected H_o at 5% for sample size 120 and rejected H_o at 10% for sample size 150. However, only the KS test does not reject H_o at both significance levels.

3.3 Discussion

Illustrate one results showed only the KS test suggested that the data set follow a standard normal distribution at 5% while other tests contradicted (i.e. large sample data set n=155). In illustrate two results, RJ and JB tests show that the data set to follow a standard normal distribution at 5% while others contradicted (small sample data set n=40). Illustrate three results showed SW, RJ and JB tests suggested that the data set follow a standard normal distribution at both level of significance (5% and 10%) while the other two (AD and KS) contradicted. The AD test shows that it does not follow a standard normal distribution when the sample size (n) is120 at 10%. Similarly, the KS test also shows that it does not follow a standard normal distribution when the sample size (n) is150 at 10% and when the sample size (n) is120 at 5%. Finally, illustrate four results, only the KS test suggested that the data set follows a standard normal at both level of significance (5% and 10%) while other tests contradicted. SW test shows that does not follow a standard normal distribution when the sample size (n) is 35 at 10%; also AD test shows that does not follow a standard normal distribution when the sample size (n) is 40 at 10%; RJ test shows that does not follow a standard normal distribution when the sample sizes (n) are 40 and 35 at 10%; while JB test shows that does not follow a standard normal distribution, when the sample size (n) is 40 at 10% and when the sample size (n) is 35 at 5%.

Note: for large sample sizes at a 5% level of significance; only the KS test detects that the simulated data do not follow a normal distribution, while for small sample sizes at a 5% level of significance only the JB test detects that the data do not follow a normal distribution.

4 Conclusion

This paper compared five different normality tests, using four illustrations (Two secondary and primary data sets). These tests were applied to different data sets; large sample with outliers, small sample without outliers, simulated large sample sizes (150, 140, 130, 130, 110 and 100) and simulated small sample sizes (40. 35, 30, 25, 20, 15 and 10). The various tests were done at 5% and 10% level of significance. The results show that for the large sample sizes at 5% level of significance; Kolmogorov-Smirnov (KS) test is a most powerful test than other tests since it detects the simulated large sample data sets do not follow a normal distribution at 5%, while for small sample sizes at 5% level of significance; the results showed the Jarque-Bera (JB) test is a most powerful test than other tests since it detects that the simulated small sample data do not follow a normal distribution at 5%.

This paper recommended the JB test for normality tests when the sample size is small and the KS test when the sample size is large at a 5% level of significance.

Competing Interests

Authors have declared that no competing interests exist.

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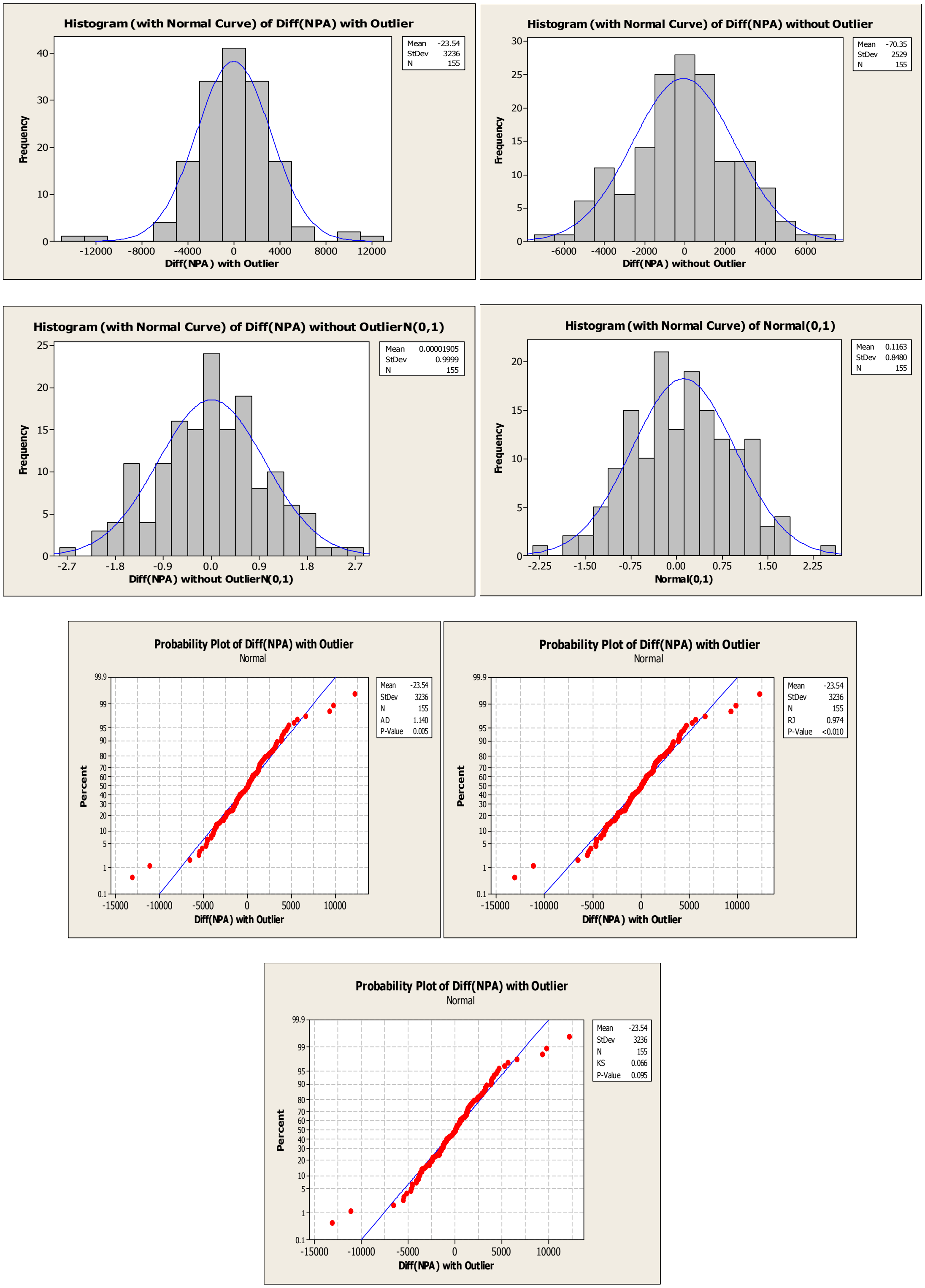
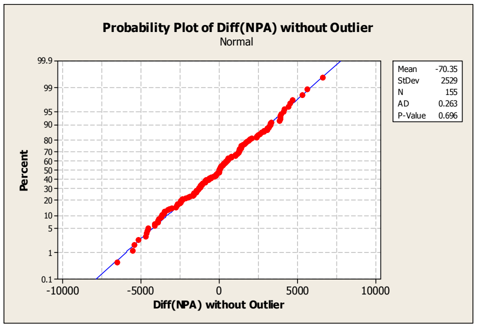
APPENDIX

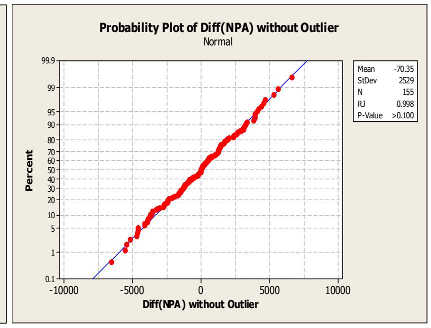


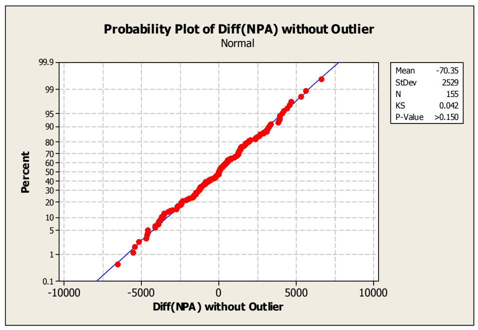
Tests of normality

	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Statistic	**Df**	**Sig.**	**Statistic**	**Df**	**Sig.**
Diff(NPA) with Outlier	.066	155	.095	.956	155	.000

a. Lilliefors Significance Correction



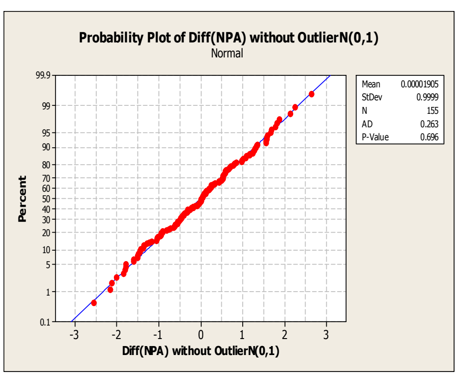


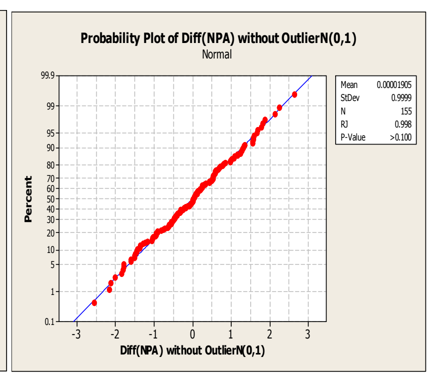


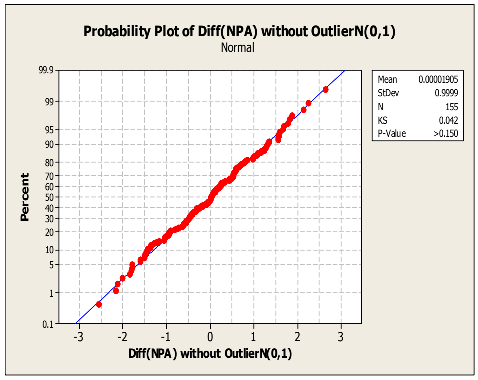
Tests of normality

	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Statistic	**Df**	**Sig.**	**Statistic**	**df**	**Sig.**
Diff(NPA) without Outlier	.043	155	.200*	.995	155	.879

a. Lilliefors Significance Correction
**. This is a lower bound of the true significance.*



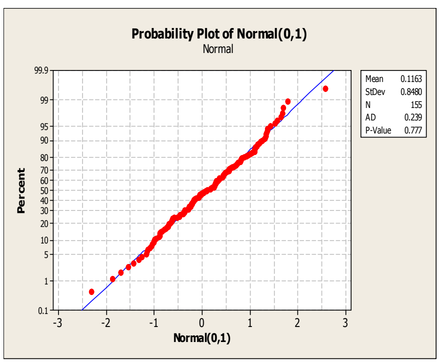


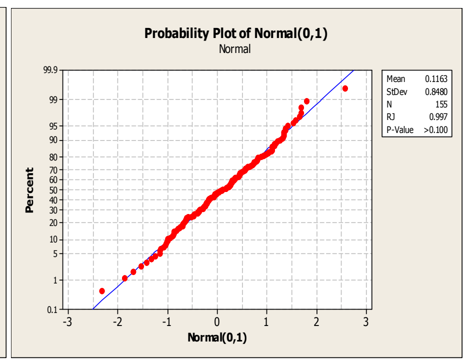


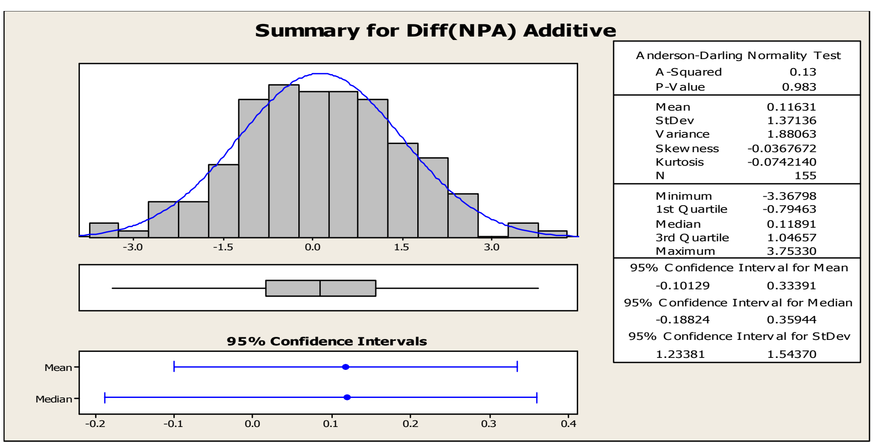
Tests of normality

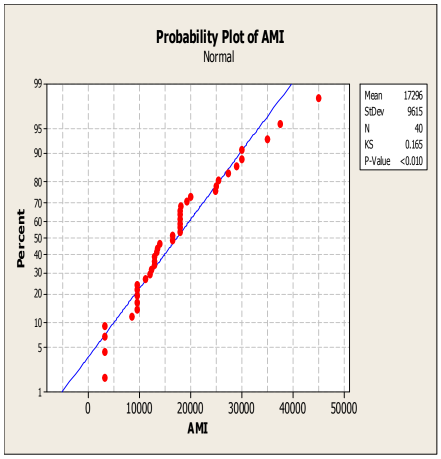
	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Statistic	Df	Sig.	Statistic	Df	Sig.
Diff(NPA) without Outlier N(0,1)	.043	155	.200*	.995	155	.879

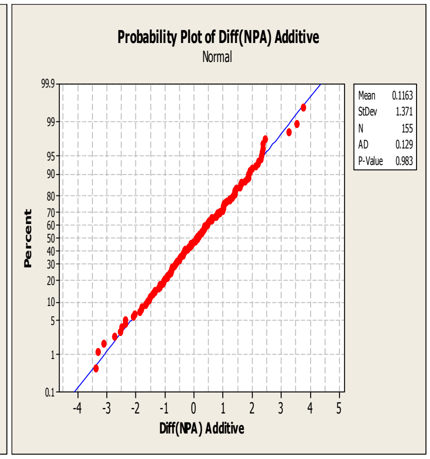
a. Lilliefors Significance Correction
**. This is a lower bound of the true significance.*

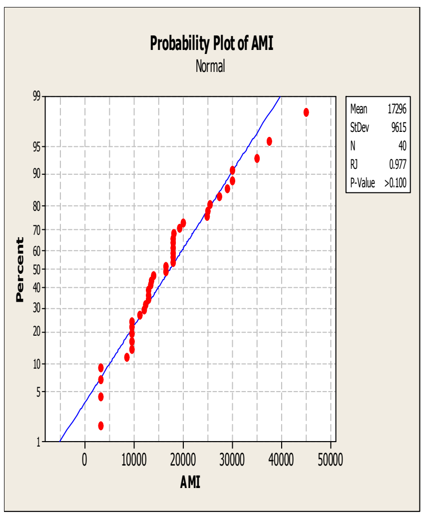


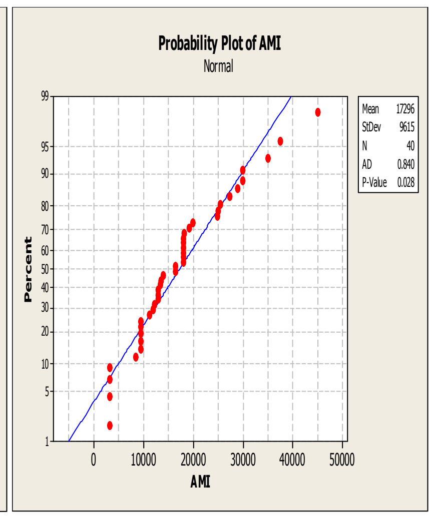












Tests of normality

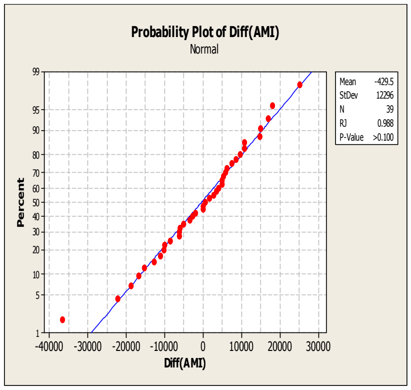
	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Statistic	**df**	**Sig.**	**Statistic**	**Df**	**Sig.**
Diff(AMI)	.078	39	.200*	.982	39	.782
AMI	.172	39	.005	.934	39	.025

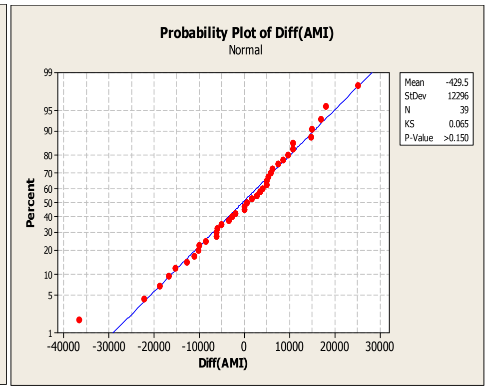
a. Lilliefors Significance Correction
**. This is a lower bound of the true significance*

Tests of normality

	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Statistic	**df**	**Sig.**	**Statistic**	**df**	**Sig.**
Diff(AMI) N(0,1)	.078	39	.200*	.982	39	.782

a. Lilliefors Significance Correction
**. This is a lower bound of the true significance*

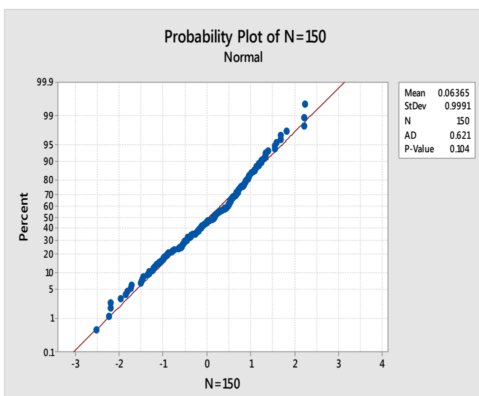


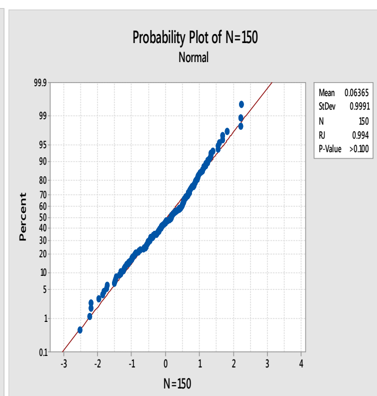


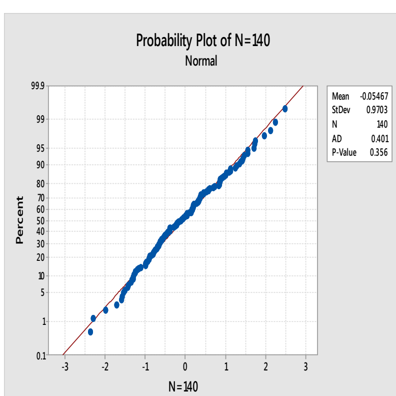
Tests of normality

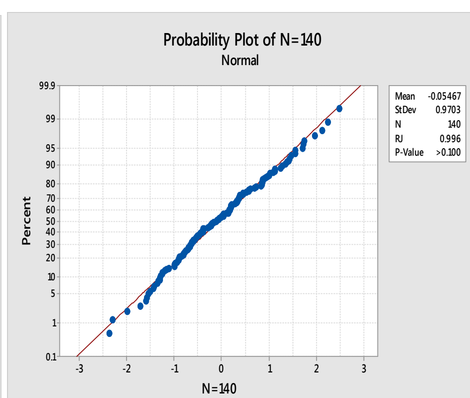
	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Statistic	Df	Sig.	Statistic	df	Sig.
N=150	.082	100	.097	.985	100	.336
N=140	.051	100	.200*	.992	100	.792
N=130	.051	100	.200*	.986	100	.371
N=120	.099	100	.017	.982	100	.180
N=110	.053	100	.200*	.992	100	.785
N=100	.040	100	.200*	.991	100	.765

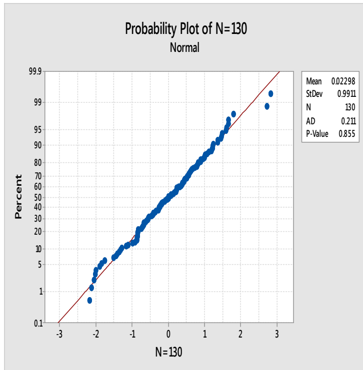
**. This is a lower bound of the true significance.*
a. Lilliefors Significance Correction

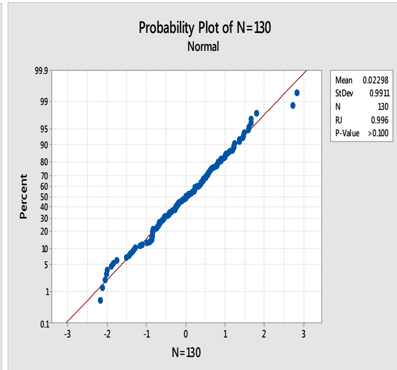


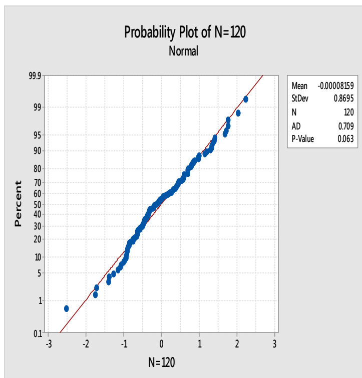


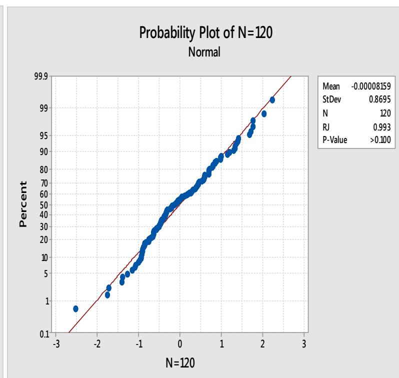


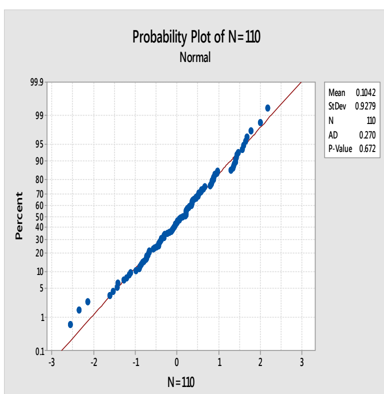


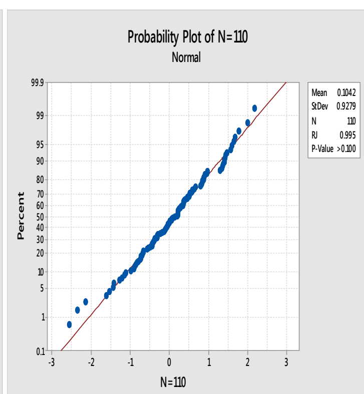

Probability Plot of N=130
Normal
Mean 0.02298
StDev 0.9911
N 130
AD 0.211
P-Value 0.855
Percent
N=130

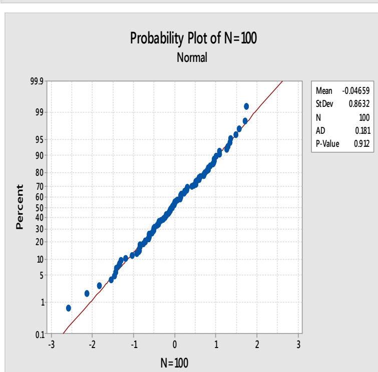

Probability Plot of N=130
Normal
Mean 0.02298
StDev 0.9911
N 130
RJ 0.996
P-Value >0.100
Percent
N=130

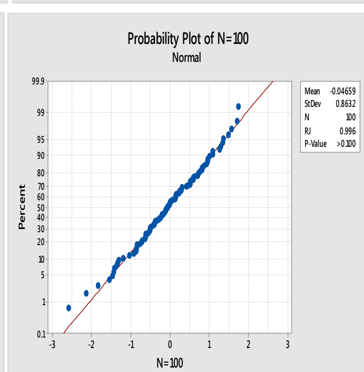

Probability Plot of N=120
Normal
Mean -0.00008159
StDev 0.8695
N 120
AD 0.709
P-Value 0.063
Percent
N=120


Probability Plot of N=120
Normal
Mean -0.00008159
StDev 0.8695
N 120
RJ 0.993
P-Value >0.100
Percent
N=120


Probability Plot of N=110
Normal
Mean 0.1042
StDev 0.9279
N 110
AD 0.270
P-Value 0.672
Percent
N=110


Probability Plot of N=110
Normal
Mean 0.1042
StDev 0.9279
N 110
RJ 0.995
P-Value >0.100
Percent
N=110

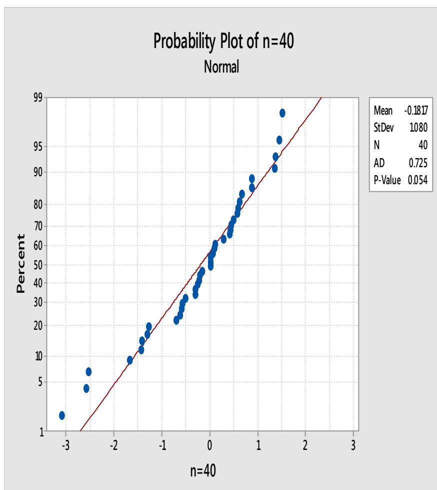

Probability Plot of N=100
Normal
Mean -0.04659
StDev 0.8632
N 100
AD 0.181
P-Value 0.912
Percent
N=100

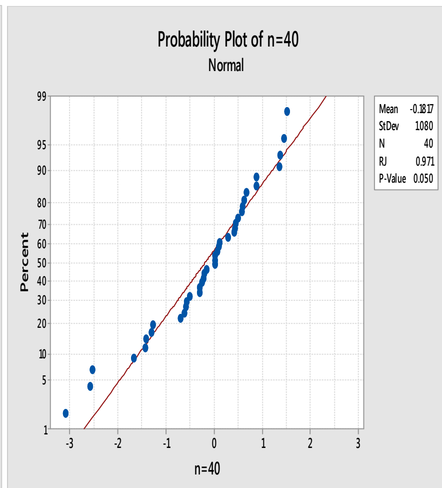

Probability Plot of N=100
Normal
Mean -0.04659
StDev 0.8632
N 100
RJ 0.996
P-Value >0.100
Percent
N=100

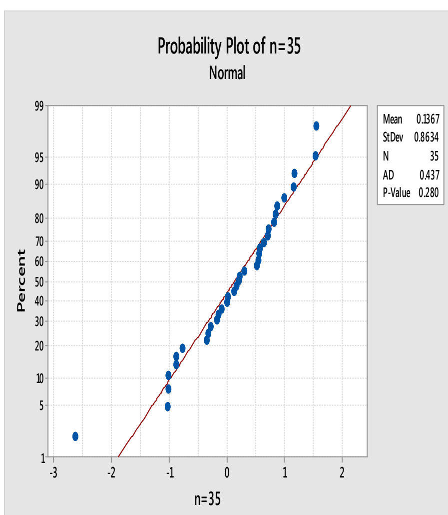
Tests of normality

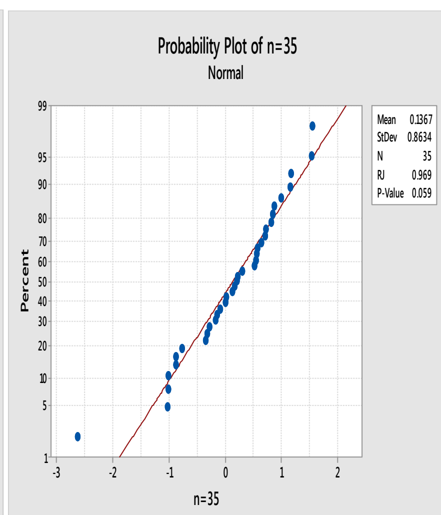
	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Statistic	**Df**	**Sig.**	**Statistic**	**df**	**Sig.**
n=40	.202	10	.200*	.957	10	.751
n=35	.193	10	.200*	.853	10	.063
n=30	.131	10	.200*	.960	10	.784
n=25	.119	10	.200*	.968	10	.868
n=20	.156	10	.200*	.940	10	.550
n=15	.130	10	.200*	.957	10	.753
n=10	.120	10	.200*	.957	10	.747

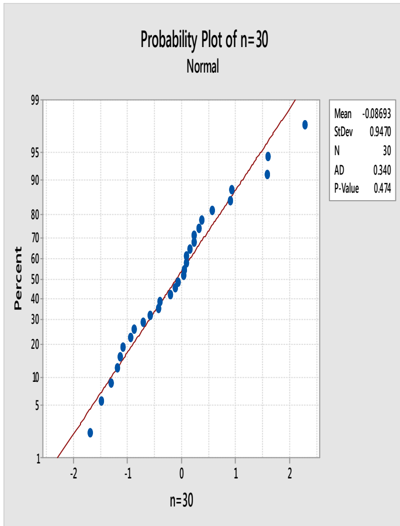
**. This is a lower bound of the true significance.*
a. Lilliefors Significance Correction

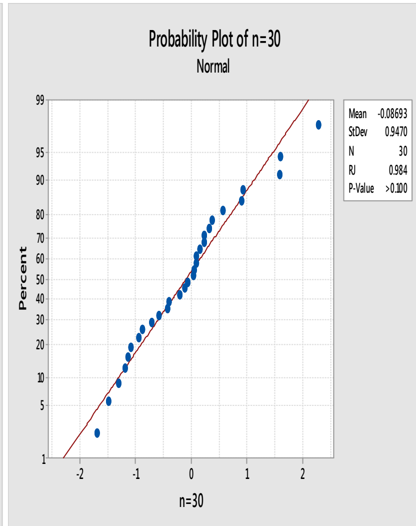


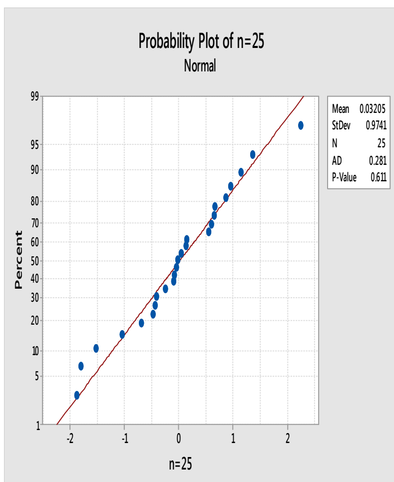


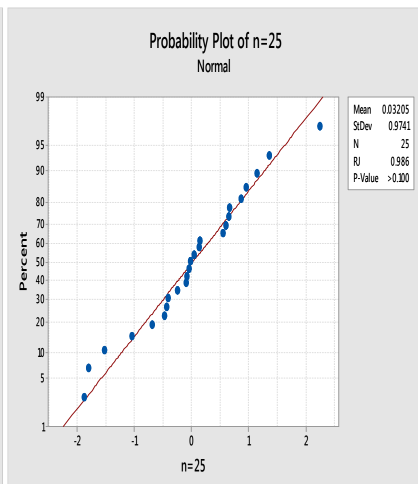





Probability Plot of n=30
Normal
Percent
n=30
Mean -0.08693
StDev 0.9470
N 30
AD 0.340
P-Value 0.474


Probability Plot of n=30
Normal
Percent
n=30
Mean -0.08693
StDev 0.9470
N 30
RJ 0.984
P-Value >0.100


Probability Plot of n=25
Normal
Percent
n=25
Mean 0.03205
StDev 0.9741
N 25
AD 0.281
P-Value 0.611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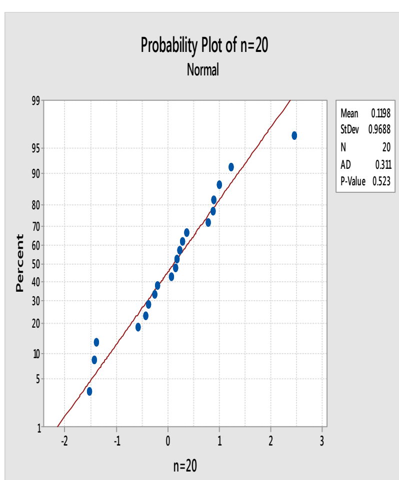
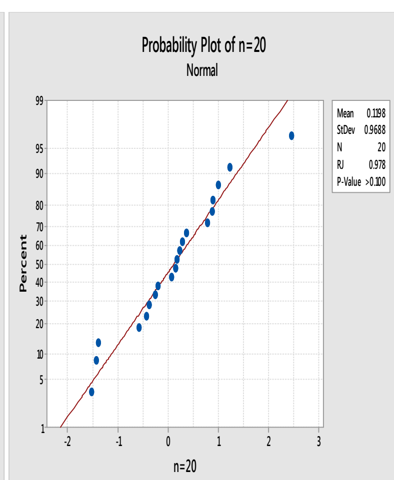
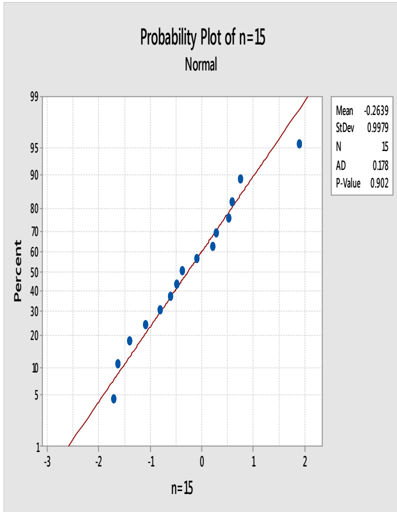
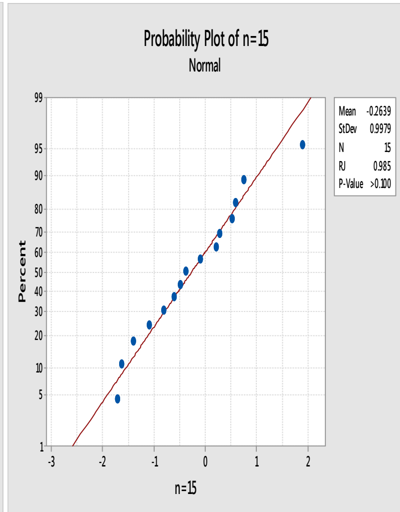
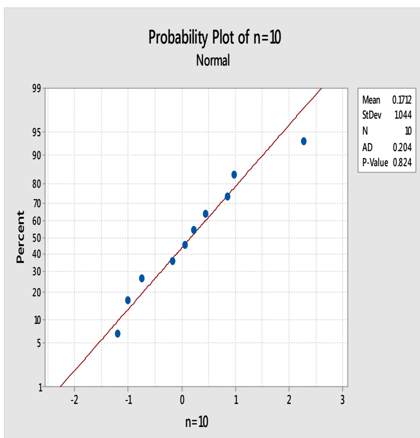
Normal
Percent
n=25
Mean 0.03205
StDev 0.9741
N 25
RJ 0.986
P-Value >0.100

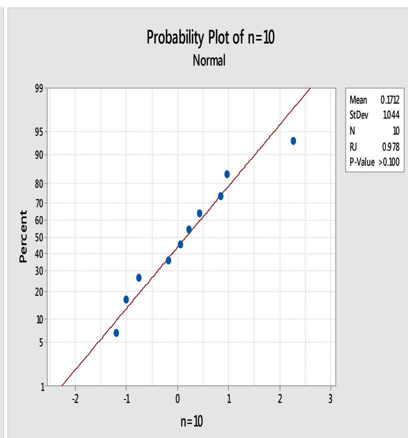

Probability Plot of n=20
Normal
Percent
n=20
Mean 0.1198
StDev 0.9688
N 20
AD 0.311
P-Value 0.523


Probability Plot of n=20
Normal
Percent
n=20
Mean 0.1198
StDev 0.9688
N 20
RJ 0.978
P-Value >0.100









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