buildings

MDPI

Article

Time Domain Nonlinear Dynamic Analysis of Vertically Loaded Tapered Pile in Layered Soils

Qiangqiang Shua [1,2], Kexing Liu [3], Jingkai Li [4] and Wenbo Tu [3,*]

[1] School of Architectural Engineering, Sichuan University of Arts and Science, Dazhou 635000, China
[2] Building Environment Engineering Technology Research Center in Dazhou, Sichuan University of Arts and Science, Dazhou 635000, China
[3] School of Transportation Engineering, East China Jiaotong University, Nanchang 330013, China
[4] China Huaxi Engineering Design & Construction Co., Ltd., Chengdu 610000, China
* Correspondence: wenbotu@ecjtu.edu.cn

Abstract: A simplified model is proposed for predicting the nonlinear dynamic response of vertically loaded tapered piles in the time domain, in which the tapered pile is divided into several frustum segments and the four-spring is used for the simulation of the soil–pile interaction. The differential equations for the tapered pile are given and solved by the finite difference method. The vertical dynamic response of a typical tapered pile is investigated, and the consistency of the computational results compared with the finite element results convincingly verifies the reliability of the proposed simplified model. Then, recommended segment numbers, considering the computational efficiency and accuracy requirements for the dynamic analysis of tapered piles, are given. And parametric studies are also carried out to investigate the effect of soil and pile parameters on the nonlinear dynamic response of the tapered pile. The results show that soil nonlinearity significantly affects the vertical dynamic characteristics of the tapered pile. And the tapered pile shows better dynamic characteristics than the cylindrical pile with the same volume and pile length. In addition, the properties of the soil along the upper part of the tapered pile have a more considerable effect on the dynamic response of the tapered pile. These results help to further improve the theory of nonlinear dynamic response analysis of tapered piles and promote its widespread application in engineering practice.

Keywords: tapered pile; time domain; vertical load; dynamic response

Citation: Shua, Q.; Liu, K.; Li, J.; Tu, W. Time Domain Nonlinear Dynamic Analysis of Vertically Loaded Tapered Pile in Layered Soils. *Buildings* **2024**, *14*, 445. https://doi.org/10.3390/buildings14020445

Academic Editors: Humberto Varum and Fabrizio Gara

Received: 26 December 2023
Revised: 23 January 2024
Accepted: 30 January 2024
Published: 6 February 2024

1. Introduction

Tapered piles are a novel type of pile that originated in the former Soviet Union in the 1970s. Compared to cylindrical piles of the same volume, calculated by the theoretical or numerical method used in engineering practices [1,2], the load-carrying capacity is increased by 0.5~2.5 times [3–6]. Due to its advantages over traditional pile foundations mentioned above, it is also used in structural foundations, slope retaining, and railway or highway weak foundation treatment projects [7–9]. Many researchers have investigated the vertical and lateral bearing capacity of the tapered pile. However, a limited understanding of the tapered pile–soil dynamic interaction mechanisms has affected the widespread use of tapered piles. Sudhendu and Ghosh [10] proposed a simplified analytical method for the dynamic characteristics of the vertically loaded tapered piles and gave the factors affecting the dimensionless stiffness and damping coefficients of the pile. Xie and Vaziri [11] established a mathematical model for the vertical harmonic vibration of tapered piles and verified it by field tests. Wu et al. [12] and Wang et al. [13] divided the tapered pile into several continuous stepped cylindrical segments and then solved the vertical vibration differential equation using the Rayleigh–Love bar theory to obtain the dynamic impedance of the pile. Nevertheless, all the above analyses were based on the assumption that the soil is a uniform elastic medium and that the soil is assumed to be an elastic material.

Actually, soil is distributed in a layered pattern and will show nonlinear characteristics when subjected to a large load. Limited studies have been conducted to calculate the nonlinear dynamic response of tapered piles considering the nonlinear interaction between the soil and the pile, and the effects of the soil stratification on the dynamic characteristics of the vertically loaded tapered pile were also not considered [14–17].

On the other hand, the tapered pile is a typical pile whose cross-section changes linearly with pile length, and the interaction between the soil and tapered pile is improved by a wedge-shaped surface, as shown in Figure 1. However, the current theoretical and analytical models mentioned above almost always treat tapered piles approximately as stepped piles, the inclined surface is treated as a vertical surface, and the effects of the taper angle are not fully considered for a dynamic analysis of tapered piles [18–20]. Hu et al. [21] provide a solution to calculate the dynamic impedance of the vertically harmonic loaded tapered pile, considering the effects of the tapered angle in accordance with the elastic dynamic Winkler theory. However, the soil was considered an elastic body, and this approach cannot be further extended to the nonlinear dynamic response analysis. This means that it is impossible to obtain the time-dependent response of the foundation, and at the same time, it is impossible to obtain the dynamic response of the foundation in the case of complex loads, such as seismic loads, wind loads, etc. Obviously, the current understanding of the bearing characteristics of tapered piles is still insufficient, which will prevent its development and application.

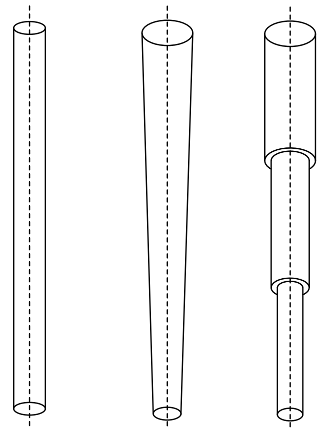


Figure 1. Illustrations of a cylindrical pile, tapered pile and stepped pile.

To address the shortcomings of the current research analysis of tapered piles under vertical loads, this paper focuses on investigating the nonlinear dynamic response of tapered piles, considering the taper angle effect. Firstly, a pile–soil interaction model based on the nonlinear Winkler dynamic theory was established, and the nonlinear vertical dynamic characteristics of the tapered pile were obtained by solving the vertical vibration differential equation using the finite difference method and the Newmark-β method. Secondly, the effect of the number of tapered pile segments on the calculation accuracy of the nonlinear dynamic response is discussed. Finally, parametric studies were conducted to explore the effect of the tapered pile and soil parameters on the dynamic characteristics of the vertically loaded tapered pile. These results help to further improve the theory of nonlinear dynamic response analysis of tapered piles and promote its widespread application in engineering practices.

2. Vertical Vibration of Tapered Piles in Layered Soils

2.1. Vertical Vibration Analysis Model

As shown in Figure 1, the vibration properties of a tapered pile subjected to vertical harmonic load in layered soil are established based on the dynamic Winkler theory [22,23], where the taper angle, pile length and pile top radius of the tapered pile are θ, L and r_0, respectively. And the assumptions below are introduced to facilitate the analysis:

(1) It is assumed that the pile is elastic, the surface of the pile is always boned to the soil, the pile deformation exists only in the *y*-*z* plane and the displacement perpendicular to the pile is neglected, as shown in Figure 2.
(2) It is assumed that the soil is distributed in layers, and the height of the pile discretization is the same as that of the soil discretization. The pile–soil interaction was simulated as a continuous series of separate soil springs and dampers distributed around the pile shaft.
(3) It is assumed that the vertical harmonic load acts on the pile top, and the vertical vibration mode of the pile also exhibits a harmonic vibration mode.
(4) It is assumed that each layer of soil is isotopically homogeneous. The soil around the pile is assumed to be a plane strain model with no forces on the surface of the soil.

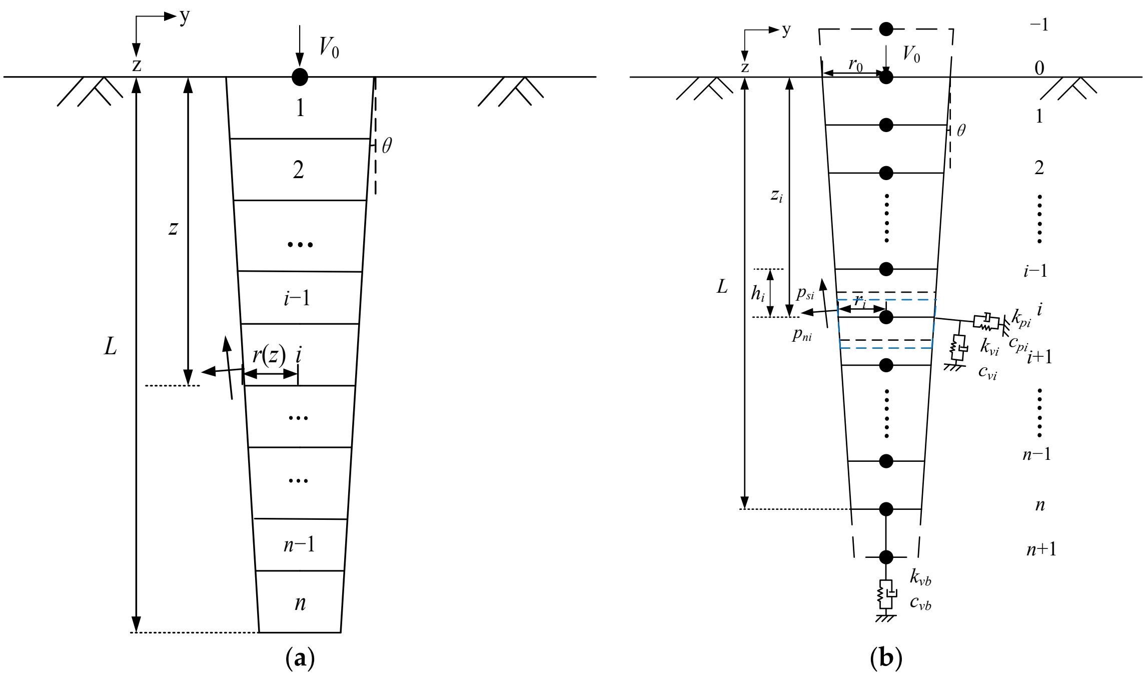


Figure 2. Vertical vibration analysis model of the tapered pile: (**a**) pile differential element; (**b**) discretized by the finite differential method.

As illustrated in Figure 2a, the tapered pile is divided into n frustum segments along the pile shaft According to the dynamic equilibrium condition, the differential equation of vertical vibration of ith segment of the tapered pile, neglecting the internal damping coefficients of the tapered pile, can be expressed as follows:

$$m(z)\frac{\partial^2 w(z,t)}{\partial t^2} + c\frac{\partial w(z,t)}{\partial t} + kw(z,t) = EA(z)\frac{\partial^2 w(z,t)}{\partial z^2} + E\frac{dw(z,t)}{dz}\frac{dA(z)}{dz} = 0 \quad (1)$$

where E is the elastic modulus of pile, k and c are the stiffness and damping coefficient of soil, $m(z)$ and $A(z)$ are pile mass per unit length and cross-sectional area at depth z, $w(z, t)$ is the vertical displacement of pile at depth z and time t.

To solve Equation (1), the equation is discretized by the finite difference method, and the model can be illustrated as in Figure 2b. The height, radius and vertical displacement of the i_{th} segment are h_i, r_i and w_i, respectively. The soil surrounding the tapered pile is modeled as a continuous distribution of springs and dampers to model the soil–pile dynamic interaction, where the springs k_{vi} and dampers c_{vi} are perpendicular to the normal direction of the pile foundation, and the spring k_{pi} and damper c_{pi} are parallel to the normal direction of the pile foundation. The tangential force p_{ti} and normal force p_{ni} along the tapered pile will be generated at the ith segment of the pile when the pile experiences a vertical dynamic load $V = V_0 e^{i\omega t}$ with circular frequency ω. This is different from the cylindrical pile due to the different form of the pile–soil contact surface. The vibration equilibrium equation at each point of a vertically loaded tapered pile can be derived at equidistant points z_i, z_{i-1} and z_{i+1} [24], as follows:

$$\frac{E_{p0}A_{p0}}{h_i{}^2}[2w_0 - 2w_1] + k_{vp0}w_0 = \frac{E_{p0}A_{p0}}{h_i{}^2}(1 - \frac{ah_i}{az+b})\frac{2h_i p_0}{E_{p0}A_{p0}}, (i = 0) \tag{2}$$

$$\frac{E_{pi}A_{pi}}{h_i{}^2}[-(1 - (\frac{ah_i}{az+b}))w_{i-1} + 2w_i - (1 + \frac{ah_i}{az+b})w_{i+1}] + k_{vpi}w_i = 0, (i = 1 \sim n-1) \tag{3}$$

$$\frac{E_{pn}A_{pn}}{h_i{}^2}[-2w_{n-1} + (2 + (1 + \frac{ah_i}{az+b})\frac{2h_i k_{bz}}{E_{pn}A_{pn}})w_n] + k_{vpn}w_n = 0,\ (i\ =\ n) \tag{4}$$

$$k_{vpi} = k_{vi}(\cos\theta)^2 + k_{pi}(\sin\theta)^2, \tag{5}$$

$$c_{vpi} = c_{vi}(\cos\theta)^2 + c_{pi}(\sin\theta)^2, \tag{6}$$

where E_{pi}, m_{pi} and A_{pi} are the elastic modulus, mass and cross-sectional area of the ith frustum pile segment, respectively. The dynamic stiffness and damping coefficients of the springs and dampers along the tapered pile shaft are to be determined as [25–27]

$$k_{vi} \approx 0.6E_{si}\left(1 + \frac{1}{2}\sqrt{a_{0i}}\right), \tag{7}$$

$$c_{vi} \approx 2\beta_{si}\frac{k_{vi}}{\omega} + \pi\rho_{si}V_{si}d_i a_{0i}^{-\frac{1}{4}}, \tag{8}$$

$$k_{pi} \approx 1.2E_{si}, \tag{9}$$

$$c_{pi} \approx 2\beta_{si}\frac{k_{pi}}{\omega} + 6\rho_{si}V_{si}d_i a_{0i}^{-\frac{1}{4}}, \tag{10}$$

where E_{si}, V_{si}, ρ_{si} and β_{si} are the soil elastic modulus, shear wave velocity, density and damping ratio of the ith soil segment. D_i is the diameter of the ith pile segment. $A_{0i} = \omega d_i / V_{si}$ is the dimensionless frequency.

The pile tip segment can be approximated as a rigid mass block embedded in an elastic foundation. Excluding the springs along the pile shaft, the dynamic stiffness and damping coefficients of the spring at the pile tip can be obtained from the vertical vibration solution given by Lysmer and Richart [28]:

$$k_{vb} = \frac{4G_b r_b}{1 - v_b}, \tag{11}$$

$$c_{vb} = \frac{3.4r_b\sqrt{G_b\rho_b}}{1 - v_b}, \tag{12}$$

where k_{vb} and c_{vb} are the stiffness and damping coefficients of soil at the pile tip, respectively. G_b, v_b and ρ_b are the soil shear modulus, Poisson's ratio and soil density at the pile tip, respectively.

Combining the finite difference method and the Newmark-β method, the dynamic response of a vertically loaded tapered pile can be derived as

$$\{\Delta w\} = \left(\frac{E_{pi}A_{pi}}{h_i{}^2}\left[I_{pl}\right] + b_0\left[M_p\right] + b_1[C_v] + \left[K_{vp}\right]\right)^{-1}\left\{\Delta\widetilde{F}_v\right\} = \left[\widetilde{K}_v\right]^{-1}\left\{\Delta\widetilde{F}_v\right\}, \tag{13}$$

$$\left\{\Delta\widetilde{F}_v\right\} = \begin{bmatrix}\Delta F_{v0} & \Delta F_{v1} & \cdots & \Delta F_{vi} & \cdots & \Delta F_{v(n-1)} & \Delta F_{vn}\end{bmatrix}^T, \tag{14}$$

$$\Delta F_{vi} = \Delta F_v + m_i[b_2\dot{w}(t) + b_3\ddot{w}(t)] + c_{vi}[b_4\dot{w}(t)], \tag{15}$$

where $\left[\widetilde{K}_v\right]$ is the vertical equivalent total stiffness matrix, $\{\Delta w\}$ and $\left\{\Delta\widetilde{F}_v\right\}$ are the increments of the vertical displacement and vertical equivalent load vectors. B_0, b_1, b_2, b_3, b_4 are the Newmark parameters, $[M_p]$, $[K_{vp}]$ and $[C_v]$ are the mass, dynamic stiffness and damping matrices of the tapered pile, respectively. $[I_{pl}]$ is the coefficient matrix generated by the computational process, given as

$$\left[I_{pl}\right] = \begin{bmatrix} 2 & -2 & & & & & \\ -(1-A_1) & 2 & -(1+A_1) & & & & \\ & & \ddots & & & & \\ & & -(1-A_i) & 2 & -(1+A_i) & & \\ & & & & \ddots & & \\ & & & & -(1-A_{n-1}) & 2 & -(1+A_{n-1}) \\ & & & & & & 2+(1+A_n)\frac{2k_{bz}}{h_i} \end{bmatrix}, \tag{16}$$

where $A_i = \frac{-\tan\theta h_i}{-\tan\theta z_i + r_0}$, z_i is the length of the ith pile node to the ground surface.

2.2. Nonlinear Analysis Model

Actually, the soil surrounding the pile will exhibit typical nonlinear characteristics when the soil is subjected to a vertical dynamic load. A hyperbolic nonlinear model with the Masing criterion was introduced to simulate the nonlinear characteristics of soil, as shown in Figure 3 [29–31].

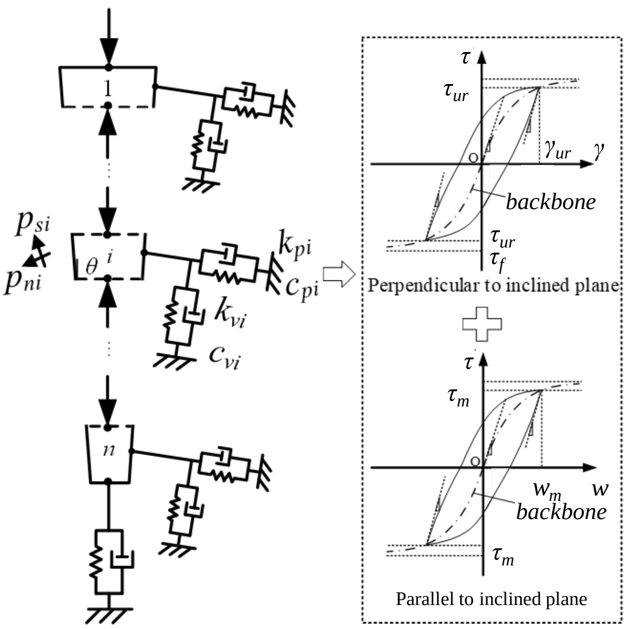


Figure 3. Nonlinear analysis model of the tapered pile.

For the soil parallel to the inclined plane of the tapered pile, the stress–displacement hyperbolic nonlinear model is given as [29,30]

$$\tau \pm \tau_m = \frac{w \pm w_m}{a + \frac{|w - w_m|}{2b}}, \tag{17}$$

where τ_m is the yield shear stress that corresponds to the displacement w_m. a is the initial flexibility factor at the soil–pile interface and b is the reciprocal of the asymptotic coefficient of the shear stress–displacement curve.

$$a = \frac{r_0 \ln|r_m / r_0|}{G_s}, \tag{18}$$

$$b = \frac{0.8}{\sigma_n' \tan\delta}, \tag{19}$$

where r_m is the radius of influence of soil displacement near the pile, δ is the effective friction angle at the soil–pile interface and σ_n' is the effective stress acting on the pile.

For the soil perpendicular to the inclined plane of the tapered pile, the stress–strain hyperbolic nonlinear model was used to model the soil nonlinearity, given as [31,32]

$$\tau_d \pm \tau_{ur} = \frac{\gamma_d \pm \gamma_{ur}}{\frac{1}{G_0} + \frac{|\gamma_d - \gamma_{ur}|}{2\tau_f}}, \tag{20}$$

$$\gamma_d = \frac{(1+v)y}{5r_0}, \tag{21}$$

where τ_d and γ_d are the current shear stress and shear strain of the soil. T_f is the shear stress at failure, and τ_{ur} is the yield shear stress corresponding to the shear strain γ_{ur}. G_0, y and r_0 are the initial small strain shear modulus of soil, the lateral displacement and the radius of the pile, respectively.

For the soil at the pile tip, the stress–strain hyperbolic nonlinear model with the Masing criterion was introduced [33,34]

$$\sigma_d \pm \sigma_m = \frac{\varepsilon_d \pm \varepsilon_m}{\frac{1}{E_0} + \frac{|\varepsilon_d - \varepsilon_m|}{2\sigma_f}}, \tag{22}$$

where σ_d and ε_d are the normal stress–strain of soil. Σ_m is the yield normal stress corresponding to the vertical strain ε_m. σ_f is the ultimate normal stress. E_0 is the initial small strain elastic modulus of soil.

Finally, in combination with the nonlinear model of the soil mentioned above, the nonlinear dynamic response of the vertically loaded tapered pile can be obtained by Equation (13) according to the initial displacement using the iterative method [35].

3. Validation and Sensitivity Analysis

3.1. Validation

The dynamic responses of vertically loaded tapered piles in homogeneous elastic soil are given in Bryden et al. [36]. The density, shear modulus and Poisson's ratio of soil are 1800 kg/m^3, 12.5 MPa and 0.25, respectively. A mass block of 5000 kg is placed on top of the pile. The elastic modulus and density of the pile are assumed to be 20 GPa and 2400 kg/m^3. The equivalent radius and pile length of the tapered pile are 0.1 m and 5.0 m. The equivalent radius is defined as

$$r_{eq}^2 = \frac{1}{3}(r_0^2 + r_0 r_b + r_b^2), \tag{23}$$

where r_b is the pile radius at the pile tip.

Figure 4 shows the stiffness and damping coefficients of the tapered pile with a taper angle θ = 1.5° calculated using the simplified proposed analysis method. The results are almost identical to those calculated by Bryden et al. [36], who give the variation curves of vertical stiffness and damping coefficients of tapered piles with dimensionless frequency a_0 under different taper angles. On the other hand, to compare the dynamic characteristics of the floating pile and end-bearing pile, the ratio of V_b/V_s is set as = 10,000 and 1 for end-bearing and floating pile, respectively [37]. On the other hand, Ghazavi [19] obtained the normalized amplitude of the tapered pile supporting a rigid foot under vertical harmonic loading and investigated the effect of different taper angles on the vertical dynamic response of the tapered pile when the equivalent radius is the same. Here, the normalized amplitude is given as

$$A_w = \frac{\omega^2}{\sqrt{\left(\frac{k_{vp}}{M_t} - \omega^2\right)^2 + \left(\frac{\omega c_{vp}}{M_t}\right)^2}}, \tag{24}$$

where M_t and ω are the footing mass and the circular frequency of the excitation load. K_{vp} and c_{vp} are the dynamic stiffness and damping of the tapered pile.

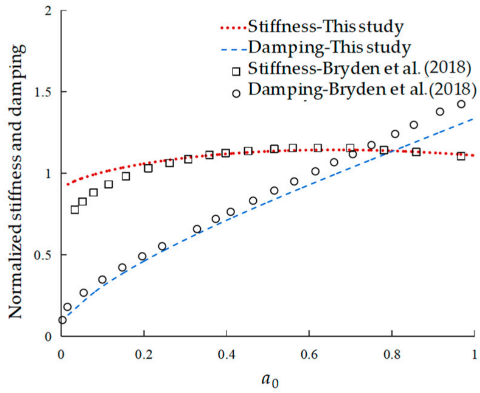
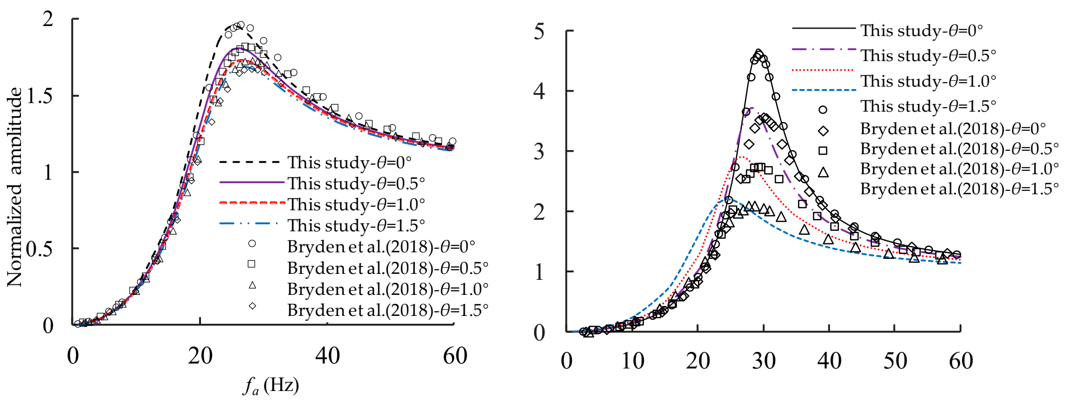


Figure 4. Comparison with the vertical dynamic stiffness and damping coefficients of the tapered pile when the tapered angle θ = 1.5° in Ref. [36].

Then, the normalized dynamic vertical responses of the tapered pile with various taper angles are compared in Figure 5. The results show that the theoretical results of this paper agree well with Bryden et al. [36], which verifies the rightness of the simplified model. It can be seen that the normalized amplitude decreases with the increase in the taper angle for the floating pile and end-bearing pile, and the resonant frequency is nearly constant for the floating pile while the resonance frequency of end-bearing piles decreases significantly. This is due to the difference in soil properties between the pile tip and the pile shaft. Compared to the dynamic response of a vertically loaded tapered pile with θ = 0°, the dynamic amplitude of the tapered pile was approximately reduced by 20% for a tapered pile with θ = 1.5°, which also shows the ability of the taper angle θ to improve the dynamic characteristics of the pile.



(**a**) (**b**)

Figure 5. Comparison of the vertical dynamic responses of tapered pile with Ref. [36]: (**a**) floating pile; (**b**) end-bearing pile.

3.2. Sensitivity Analysis

The calculation accuracy of the finite difference method is closely related to the number of pile frustum segments, namely the length of the calculation element [38,39]. Sensitivity analysis was performed for typically tapered piles with length-to-radius ratios ranging from L/r_b = 30 to 70 and 110. The dynamic harmonic load V_0 is set at 100 kN. The parameters of the tapered pile and soil are listed in Table 1 [40,41].

Table 1. Properties of the tapered pile for different slenderness ratios.

	Parameter	Value
Tapered pile	Pile length L	2.5
	Equivalent radius r_{eq}	0.1 m
	Elastic modulus E_p	20 GPa
	Density ρ_p	2400 kg/m^3
Soil	Shear wave velocity V_s	82.5 m/s
	Shear modulus G_s	12.5 MPa
	Density ρ_s	1800 kg/m^3
	Poisson's ratio v	0.25

Figure 6 shows the time history curves of vertical displacements of tapered piles at different length-to-diameter ratios. It can be found that the dynamic response of the tapered pile will tend to a stable value as the number of pile segments n increases. Therefore, for the tapered pile with L/r_b = 30–110, the accuracy requirements and computational efficiency can be met when n is taken as 200 [42,43]. Comparing the vertical displacements under different L/r_b ratios, it can be seen that when the radius of the pile tip is the same, the longer the pile length is, the smaller the vertical dynamic response of the pile is, and the resonance frequency increases with the increase in the L/r_b ratio. This is due to the vertical stiffness of the pile increasing with the increase in pile length, which means that the ability to resist vertical deformation is better. In the meantime, there is a typical shift in the displacement at the first cycle, which is also consistent with the results of El Naggar and Bentley [44]. The reason may be attributed to the increased nonlinearity of the soil and the large iterative time step in the dynamic analysis process.

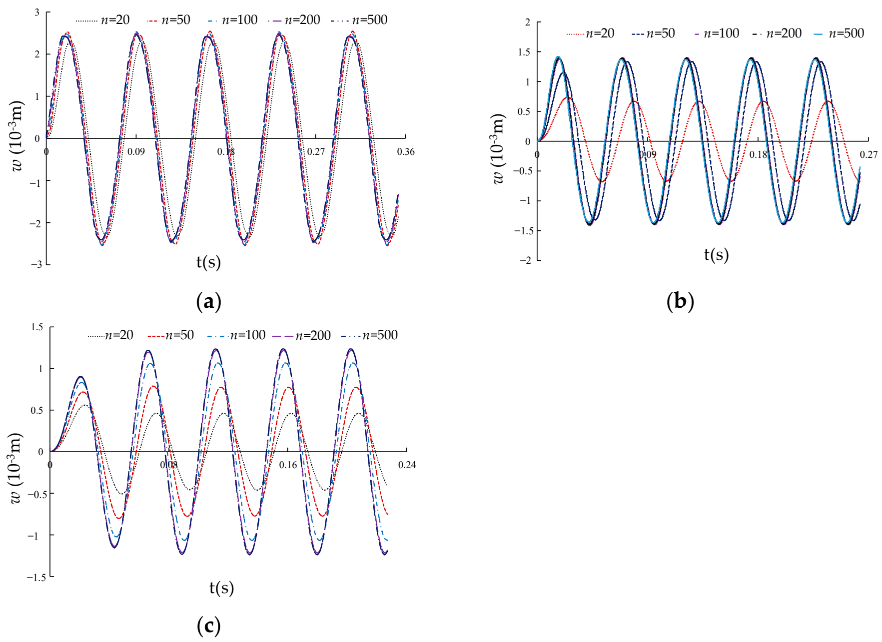


Figure 6. The time history curves of vertical displacements of tapered piles for different L/r_b ratios at pile top: (**a**) L/r_b = 30 (resonant frequency f_a = 14 Hz); (**b**) L/r_b = 70 (resonant frequency f_a = 19 Hz); (**c**) L/r_b = 110 (resonant frequency f_a = 22 Hz).

4. Parameter Discussion

4.1. Effect of Taper Angle on the Nonlinear Dynamic Response of the Tapered Pile

The taper angle is the main factor affecting the dynamic response of the tapered pile [36,45]. In this section, the dynamic responses of tapered piles with four different taper angles, 0°, 1°, 2° and 3°, are calculated, and the other parameters are the same as in Table 1.

Figure 7 shows the nonlinear dynamic response of a vertically loaded tapered pile under different taper angles. The results show that the resonance frequency of the tapered pile increases as the taper angle increases, and the nonlinear dynamic responses of the tapered pile in the vertical direction increase with a decreasing taper angle, which is consistent with the static bearing characteristics. The same pattern can be seen in the hysteresis curves of loading–displacement. However, since the reasonable selection of the pile parameters of tapered piles is beneficial to the reduction in construction costs, the dynamic response characteristics of vertically loaded tapered piles are of more concern when the pile volume is the same.

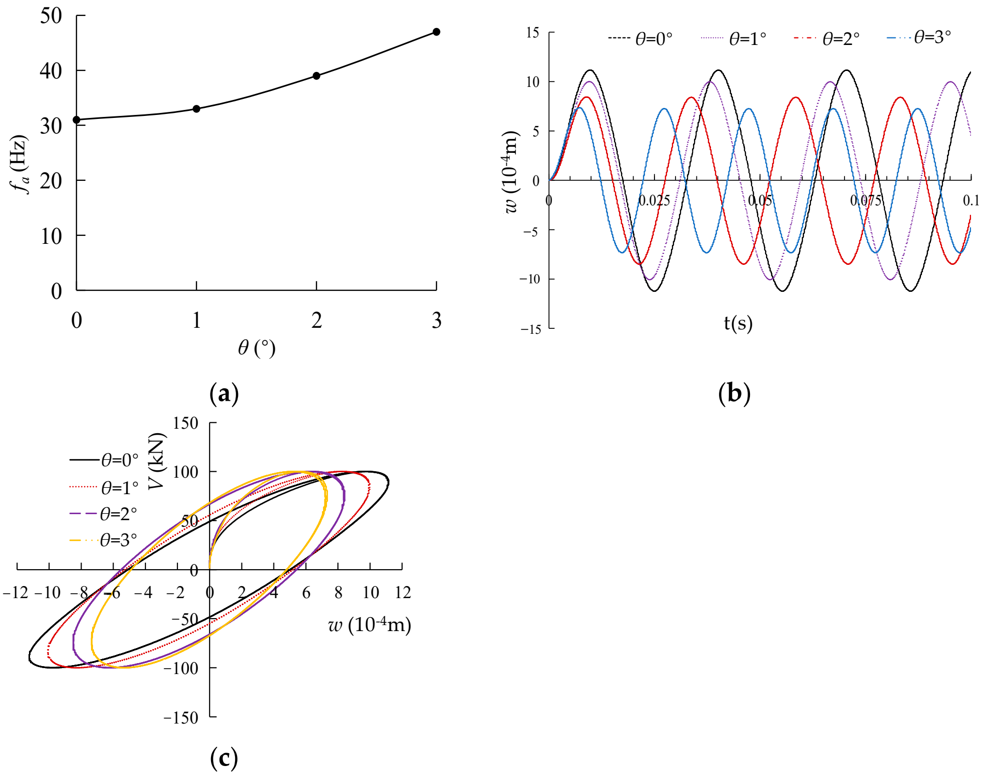


Figure 7. Nonlinear dynamic characteristics of the vertically loaded tapered pile for different taper angles with constant pile tip radius and pile length: (**a**) resonant frequency; (**b**) time history curves of vertical displacement at pile top; (**c**) hysteresis curves of loading and vertical displacement.

In order to keep the pile volume constant, two different cases of varying the diameter of the tapered pile and varying the length of the tapered pile are considered. Therefore, the resonant frequency, time history curves and hysteresis curves of the constant volume taper piles under various taper angles are shown in Figures 8 and 9. The results show that the nonlinear dynamic response of the vertically loaded tapered pile increases with a decreasing taper angle. And it can also be found that the resonance frequency of the tapered pile increases and the dynamic response of the constant volume tapered pile decreases more significantly when keeping the pile tip radius constant as compared to keeping the pile length constant. It is shown that better vertical dynamic performance can be obtained when the tapered pile keeps the radius of the pile tip constant while varying the length of the pile. This phenomenon was also found in the hysteresis curves of loading–displacement. With the increase in the taper angle, the slope of the hysteresis loop gradually increases and the amplitude of the vertical dynamic response gradually decreases. The area and shape of the hysteresis loop also tend to be more and more elliptical, which indicates that the nonlinear influence between the pile and soil is gradually reduced. This can provide engineering guidance for the design of tapered piles. It is worth noting that, although the larger the taper angle, the smaller the dynamic response of the vertically loaded tapered pile, the constructability of the pile should also be considered in engineering practice.

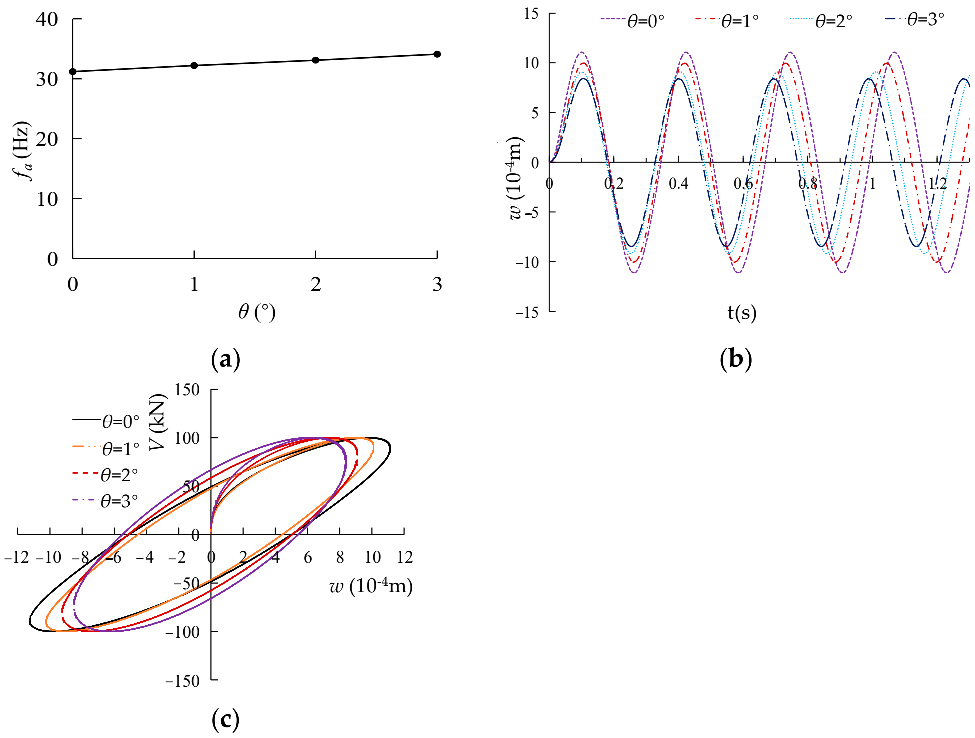


Figure 8. Nonlinear dynamic characteristics of the vertically loaded tapered pile for different taper angles with constant volume and pile tip radius: (**a**) resonant frequency; (**b**) time history curves of vertical displacement at pile top; (**c**) hysteresis curves of loading and vertical displacement.

4.2. Effect of Soil Elastic Modulus on the Nonlinear Dynamic Response of the Tapered Pile

The soil properties are also important factors influencing the dynamic behavior of tapered piles [46,47]. Three different pile–soil elastic modulus ratios, E_p/E_s = 500, 1000 and 2000, are considered for further analysis of the effect of soil parameters on the nonlinear dynamic response of the vertically loaded tapered pile. The taper angle of the pile θ is set at 1°, and the other parameters can be found in Table 1.

The resonant frequency, time history curves and hysteresis curves of loading and vertical displacement of the tapered piles with different elastic modulus ratios E_p/E_s are depicted in Figure 10. The results show that the nonlinear dynamic response of the vertically loaded tapered pile increases as the elastic modulus ratio E_p/E_s increases. The amplitude of the dynamic response increases by 267% when the elastic modulus ratios E_p/E_s are increased from 500 to 2000. This is mainly attributed to the fact that the decrease in soil modulus surrounding the pile leads to a decrease in the frictional resistance of the tapered pile. The hysteresis curve shows a counterclockwise inclination with an increase in soil modulus. This is because the increase in the soil modulus leads to an increase in the soil constraint on the pile, which also leads to a reduction in the degree of nonlinearity between the tapered pile and the soil.

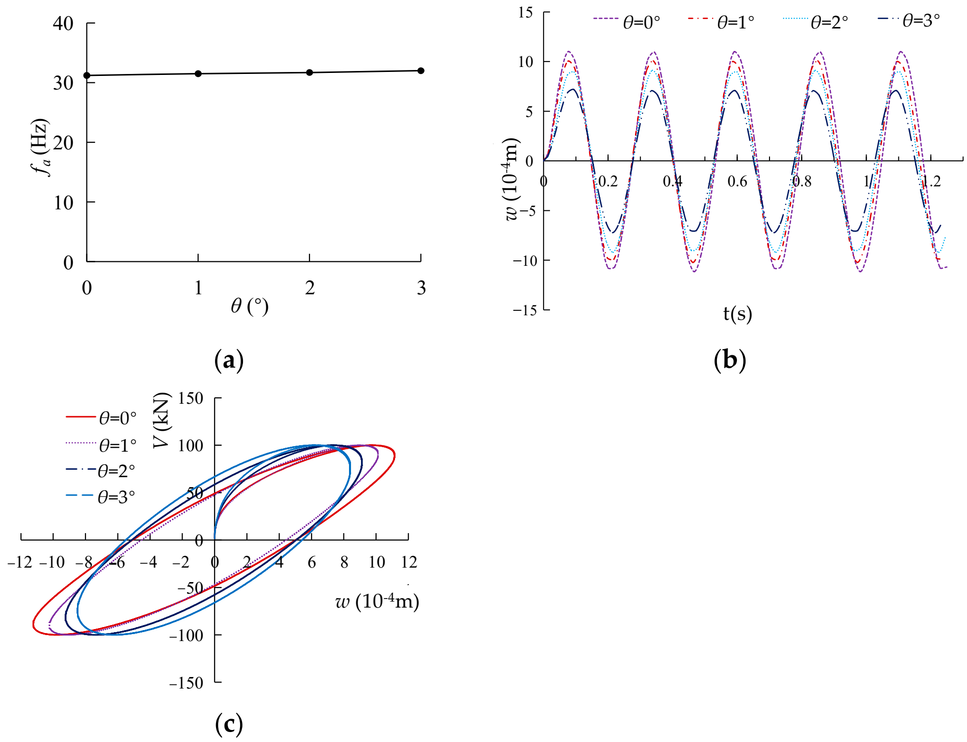


Figure 9. Nonlinear dynamic characteristics of the vertically loaded tapered pile for different taper angles with constant volume and pile length: (**a**) resonant frequency; (**b**) time history curves of vertical displacement at pile top; (**c**) hysteresis curves of loading and vertical displacement.

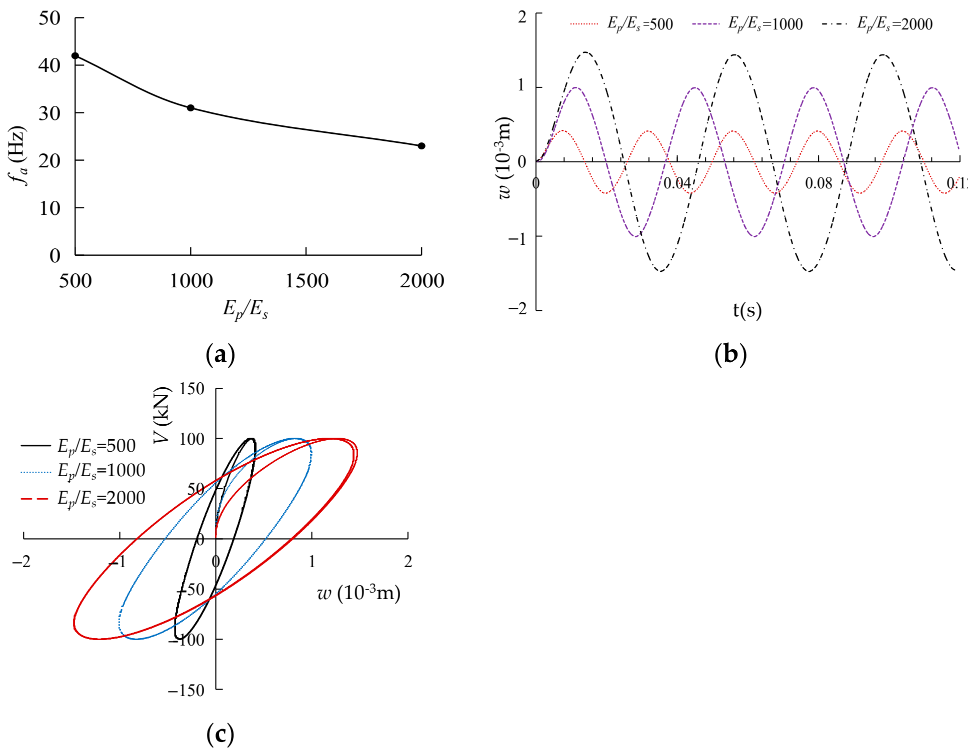
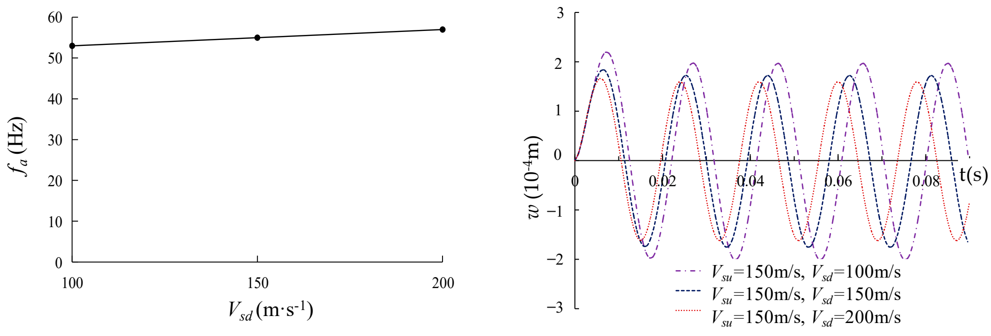


Figure 10. Nonlinear dynamic characteristics of the vertically loaded tapered pile at different pile–soil modulus ratios: (**a**) resonant frequency; (**b**) time history curves of vertical displacement at pile top; (**c**) hysteresis curves of loading and vertical displacement.

4.3. Effect of Soil Stratification on the Nonlinear Dynamic Response of the Tapered Pile

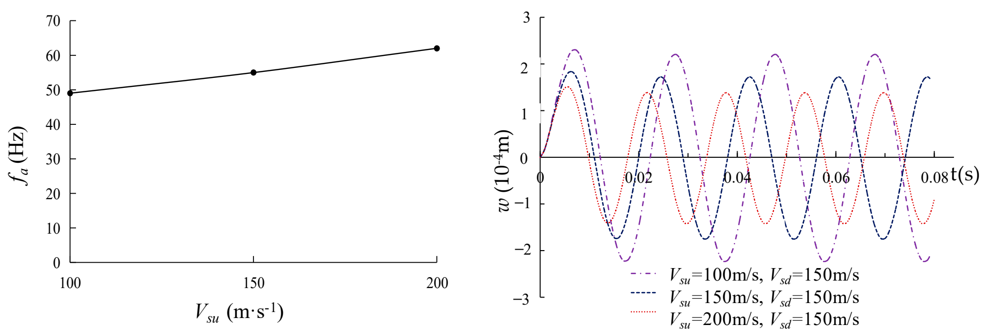
It should be noted that the former analysis of the pile is considered to be a foundation embedded in a uniform elastic half-space, while the soil in engineering practice shows layer distribution [48,49]. To clarify the effect of soil stratification on the dynamic response of the tapered pile, both two and three soil layers are discussed in this section.

Considering the soil is divided into two layers: the shear velocity of the upper soil layer V_{su} = 150 m/s remains unchanged, and the shear velocity of the lower soil layer V_{sd} gradually increases from 100 m/s to 200 m/s; the shear velocity of the lower soil layer V_{sd} = 150 m/s remains unchanged, and the shear velocity of the upper soil layer V_{su} gradually increases from 100 m/s to 200 m/s. The taper angle of the pile θ is set at 1°, and the other parameters are the same as in Table 1. The resonant frequency and time history curves of the constant volume tapered piles when considering the soil stratification are shown in Figures 11 and 12. The results show that the resonant frequency of the tapered pile increases as the shear velocity of the upper and lower soil layers increases, while the vertical dynamic response of the tapered pile decreases. Obviously, the increase in the shear wave velocity of the upper and lower soil layers will lead to an increasing ability of the tapered pile to resist vertical deformation. However, the changes in dynamic properties caused by changing the properties of the upper soil layer are more obvious, which indicates that the upper soil properties play an important role in the nonlinear dynamic response of the vertically loaded tapered pile.



(**a**) (**b**)

Figure 11. Nonlinear dynamic characteristics of the vertically loaded tapered pile at different shear wave velocities of lower soil: (**a**) resonant frequency; (**b**) time history curves of vertical displacement at pile top.



(**a**) (**b**)

Figure 12. Nonlinear dynamic characteristics of the vertically loaded tapered pile at different shear wave velocities of upper soil: (**a**) resonant frequency; (**b**) time history curves of vertical displacement at pile top.

To further analyze the influence of soil stratification, soil divided into three layers is considered: the shear velocity of the upper and lower soil layers $V_{su} = V_{sd}$ = 150 m/s remains unchanged, and the shear velocity of the middle soil layer V_{sm} gradually increases from 100 m/s to 200 m/s. The nonlinear dynamic response of the vertically loaded tapered pile is shown in Figure 13. It is found that the vertical displacement at the top of the tapered pile decreases gradually as the shear modulus of the middle soil layer increases, which also implies that the overall dynamic stiffness of the vertically loaded tapered pile is increasing. However, the change in resonant frequency and vertical displacement of the pile top is not significant, which is similar to changing the properties of the lower soil.

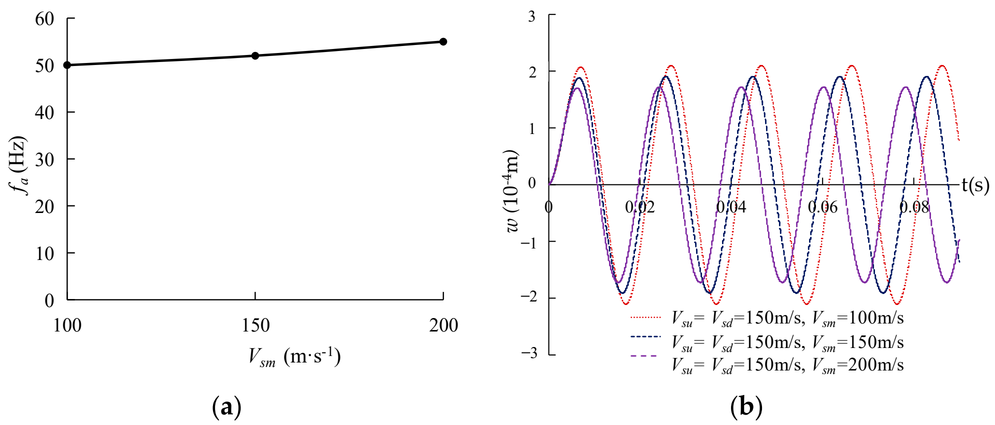


Figure 13. Nonlinear dynamic characteristics of the vertically loaded tapered pile at different shear wave velocities of middle soil: (**a**) resonant frequency; (**b**) time history curves of vertical displacement at pile top.

5. Conclusions

In this paper, the dynamic responses of the vertically loaded tapered piles are investigated based on the nonlinear dynamic Winkler model with six springs, and the influences of pile and soil parameters on the dynamic responses of vertically loaded tapered piles are discussed. The main conclusions are drawn as follows:

(1) The simplified analytical model proposed in this study can effectively obtain the nonlinear dynamic properties of the vertically loaded tapered piles, and it can ensure high computational accuracy when the pile–soil segment n is equal to 200, which also greatly reduces the computational cost.
(2) The taper angle and soil elastic modulus are the principal factors affecting the nonlinear dynamic response of vertically loaded tapered piles. The dynamic response of the tapered pile decreases with an increase in the taper angle or a decrease in the pile–soil elastic modulus ratio.
(3) For the constant volume tapered pile, the vertical dynamic response decreases more significantly when keeping the pile end radius constant as compared to when keeping the pile length constant.
(4) The soil stratification has a great influence on the nonlinear dynamic characteristics of vertically loaded tapered piles, especially the properties of the upper soil layer along the pile shaft.

Author Contributions: Conceptualization, W.T.; Methodology, W.T.; Software, Q.S. and K.L.; Validation, K.L., J.L. and W.T.; Formal analysis, K.L.; Data curation, W.T.; Writing—original draft preparation, Q.S., K.L. and J.L.; Writing—review and editing, W.T.; Visualization, J.L. and W.T.; Supervision, W.T.; Funding acquisition, Q.S. and W.T. All authors have read and agreed to the published version of the manuscript.

Funding: This work was financially supported by the National Natural Science Foundation of China (52268069), the Open Project of the Building Environment Engineering Technology Research Center in Dazhou (SDJ2022ZC-02) and Jiangxi Provincial Cultivation Program for Academic and Technical Leaders of Major Subjects (20225BCJ23011). These supports are gratefully acknowledged.

Data Availability Statement: Data are available upon reasonable request.

Conflicts of Interest: Author Jingkai Li was employed by the company China Huaxi Engineering Design & Construction Co., Ltd. The remaining authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

References

1. Conte, E.; Pugliese, L.; Troncone, A.; Vena, M. A simple approach for evaluating the bearing capacity of piles subjected to inclined loads. *Int. J. Geomech.* **2021**, *21*, 04021224. [CrossRef]
2. Tamboura, H.H.; Isobe, K.; Ohtsuka, S. End bearing capacity of a single incompletely end-supported pile based on the rigid plastic finite element method with non-linear strength property against confining stress. *Soils Found.* **2022**, *62*, 101182. [CrossRef]
3. Wei, J.; El Naggar, M.H. Experimental study of axial behaviour of tapered piles. *Can. Geotech. J.* **1998**, *35*, 641–654. [CrossRef]
4. Liu, J.; Wang, Z.H. Experimental study on the bearing capacity of wedge pile. *J. Tianjin Univ. (Sci. Technol.)* **2002**, *35*, 257–260. (In Chinese)
5. Jiang, J.P.; Gao, G.Y.; Gu, B.H. Comparison of belled pile, tapered pile and equal diameter pile. *Chin. J. Geotech. Eng.* **2003**, *25*, 764–766. (In Chinese)
6. El Naggar, M.H.; Sakr, M. Cyclic response of axially loaded tapered piles. *Int. J. Phys. Model. Geotech.* **2002**, *2*, 1–12.
7. Hataf, N.; Shafaghat, A. Optimizing the bearing capacity of tapered piles in realistic scale using 3D finite element method. *Geotech. Geol. Eng.* **2005**, *33*, 1465–1473. [CrossRef]
8. Vali, R.; Mehrinejad Khotbehsara, E.; Saberian, M.; Li, J.; Mehrinejad, M.; Jahandariet, S. A three-dimensional numerical comparison of bearing capacity and settlement of tapered and under-reamed piles. *Int. J. Geotech. Eng.* **2019**, *13*, 236–248. [CrossRef]
9. Shafaghat, A.; Khabbaz, H. Recent advances and past discoveries on tapered pile foundations: A review. *Geomech. Geoengin.* **2022**, *17*, 455–484. [CrossRef]
10. Sudhendu, S.; Ghosh, D.P. Vertical vibration of tapered piles. *J. Geotech. Eng. ASCE* **1986**, *112*, 290–302.
11. Xie, J.; Vaziri, H.H. Vertical vibration of nonuniform piles. *J. Eng. Mech. ASCE* **1991**, *117*, 1105–1118. [CrossRef]
12. Wu, W.B.; Wang, K.H.; Dou, B. Vertical dynamic response of a viscoelastic tapered pile embedded in layered foundation. *J. Vib. Shock* **2013**, *32*, 120–127. (In Chinese)
13. Wang, K.H.; Tong, W.F.; Xiao, C.; Wu, B.J. Study on dynamic response of tapered pile and model test. *J. Hunan Univ. (Nat. Sci.)* **2019**, *46*, 94–102. (In Chinese)
14. Michaelides, O.; Gazetas, G.; Bouckovalas, G.; Chrysikou, E. Approximate non-linear dynamic axial response of piles. *Géotechnique* **1998**, *48*, 33–53. [CrossRef]
15. Singh, S.; Patra, N.R. Free and forced vibration analyses of tapered piles under axial harmonic Loads. *Int. J. Geomech.* **2021**, *21*, 04021023. [CrossRef]
16. Padrón, L.A.; Aznárez, J.J.; Maeso, O. Dynamic analysis of piled foundations in stratified soils by a BEM–FEM model. *Soil. Dyn. Earthq. Eng.* **2008**, *28*, 333–346. [CrossRef]
17. He, C.; Zhou, S.H.; Guo, P.J.; Di, H.G.; Zhang, X.H. Analytical model for vibration prediction of two parallel tunnels in a full-space. *J. Sound Vib.* **2018**, *423*, 306–321. [CrossRef]
18. Liu, J.; He, J.; Wu, Y.P.; Yang, Q.G. Load transfer behaviour of a tapered rigid pile. *Géotechnique* **2012**, *62*, 649–652. [CrossRef]
19. Ghazavi, M. Analysis of kinematic seismic response of tapered piles. *Geotech. Geol. Eng.* **2007**, *25*, 37–44. [CrossRef]
20. Wang, J.; Zhou, D.; Zhang, Y.G.; Cai, W. Vertical impedance of a tapered pile in inhomogeneous saturated soil described by fractional viscoelastic model. *Appl. Math. Model.* **2019**, *75*, 88–100. [CrossRef]
21. Hu, J.; Tu, W.B.; Gu, X.Q. A simple approach for the dynamic analysis of a circular tapered pile under axial harmonic vibration. *Buildings* **2023**, *13*, 999. [CrossRef]
22. Huang, M.S.; Tu, W.B.; Gu, X.Q. Time domain nonlinear lateral response of dynamically loaded composite caisson-piles foundations in layered cohesive soils. *Soil. Dyn. Earthq. Eng.* **2018**, *106*, 113–130. [CrossRef]
23. El Naggar, M.H.; Novak, M. Non-linear model for dynamic axial pile response. *J. Geotech. Eng.* **1994**, *120*, 308–329. [CrossRef]
24. Zhang, Y. A finite difference method for fractional partial differential equation. *Appl. Math. Comput.* **2009**, *215*, 524–529. [CrossRef]
25. Gazetas, G.; Makris, N. Dynamic pile-soil-pile interaction. Part I: Analysis of axial vibration. *Earthq. Eng. Struct. Dyn.* **1991**, *20*, 115–132. [CrossRef]
26. Makris, N.; Gazetas, G. Dynamic pile-soil-pile interaction. Part II: Lateral and seismic response. *Earthq. Eng. Struct. Dyn.* **1992**, *21*, 145–162. [CrossRef]
27. Gazetas, G.; Fan, K.; Kaynia, A. Dynamic response of pile groups with different configurations. *Soil Dyn. Earthq. Eng.* **1993**, *12*, 239–257. [CrossRef]
28. Lysmer, J.; Richart Jr, F.E. Dynamic response of footings to vertical loading. *J. Soil Mech. Found. Div.* **1966**, *92*, 65–91. [CrossRef]
29. Hardin, B.O.; Drnevich, V.P. Shear modulus and damping in soils: Design equations and curves. *J. Soil Mech. Found. Div.* **1972**, *98*, 667–692. [CrossRef]

30. Zhang, Q.Q.; Zhang, Z.M.; He, J.Y. A simplified approach for settlement analysis of single pile and pile groups considering interaction between identical piles in multilayered soils. *Comput. Geotech.* **2010**, *37*, 969–976. [CrossRef]
31. Tu, W.B.; Huang, M.S.; Gu, X.Q.; Chen, H.P.; Liu, Z.H. Experimental and analytical investigations on nonlinear dynamic response of caisson-pile foundations under horizontal excitation. *Ocean Eng.* **2020**, *208*, 107431. [CrossRef]
32. Kagawa, T.; Kraft, L.M. Lateral load-deflection relationships of piles subjected to dynamic loadings. *Soils Found.* **1980**, *20*, 19–36. [CrossRef] [PubMed]
33. Tu, W.B.; Gu, X.Q.; Chen, H.P.; Fang, T.; Geng, D.X. Time domain nonlinear kinematic seismic response of composite caisson-piles foundation for bridge in deep water. *Ocean Eng.* **2021**, *235*, 109398. [CrossRef]
34. Yu, J.; Huang, M.S.; Li, S.; Leung, C.F. Load-displacement and upper-bound solutions of a loaded laterally pile in clay based on a total-displacement-loading EMSD method. *Comput. Geotech.* **2017**, *83*, 64–76. [CrossRef]
35. Daftardar-Gejji, V.; Jafari, H. An iterative method for solving nonlinear functional equations. *J. Math. Anal. Appl.* **2006**, *316*, 753–763. [CrossRef]
36. Bryden, C.; Arjomandi, K.; Valsangkar, A. Dynamic Axial Stiffness and Damping Parameters of Tapered Piles. *Int. J. Geomech.* **2018**, *18*, 06018014. [CrossRef]
37. Ghazavi, M. Response of tapered piles to axial harmonic loading. *Can. Geotech. J.* **2008**, *45*, 1622–1628. [CrossRef]
38. Troncone, A.; Pugliese, L.; Parise, A.; Conte, E. A simple method to reduce mesh dependency in modelling landslides involving brittle soils. *Géotech. Lett.* **2022**, *12*, 167–173. [CrossRef]
39. Baek, J.; Schlinkman, R.T.; Beckwith, F.N.; Chen, J.S. A deformation-dependent coupled Lagrangian/semi-Lagrangian meshfree hydromechanical formulation for landslide modeling. *Adv. Model. Simul. Eng. Sci.* **2022**, *9*, 1–35. [CrossRef]
40. Dehghanpoor, A.; Ghazavi, M. Response of tapered piles under lateral harmonic vibrations. *Geomate J.* **2012**, *2*, 261–265. [CrossRef]
41. Singh, S.; Patra, N.R. Lateral dynamic response of tapered pile embedded in a cross-anisotropic medium. *J. Earthq. Eng.* **2022**, *26*, 5826–5847. [CrossRef]
42. Cai, Y.Y.; Yu, J.; Zhen, C.T.; Qi, Z.B.; Song, B.X. Analytical solution for longitudinal dynamic complex impedance of tapered pile. *Chin. J. Geotech. Eng.* **2011**, *33*, 392–398. (In Chinese)
43. Wu, W.B.; Wang, K.H.; Wu, D.H.; Ma, B.N. Study of dynamic longitudinal impedance of tapered pile considering lateral inertial effect. *Chin. J. Rock Mech. Eng.* **2011**, *30*, 3618–3625. (In Chinese)
44. El Naggar, M.H.; Bentley, K.J. Dynamic analysis for laterally loaded piles and dynamic p-y curves. *Can. Geotech. J.* **2000**, *37*, 1166–1183. [CrossRef]
45. Gao, L.; Wang, K.H.; Xiao, S.; Wu, J.T.; Wang, N. Vertical impedance of tapered piles considering the vertical reaction of surrounding soil and construction disturbance. *Mar. Georesour. Geotechnol.* **2017**, *35*, 1068–1076. [CrossRef]
46. Kodikara, J.K.; Moore, I.D. Axial response of tapered piles in cohesive frictional ground. *J. Geotech. Eng.* **1993**, *119*, 675–693. [CrossRef]
47. Khan, M.K.; El Naggar, M.H.; Elkasabgy, M. Compression testing and analysis of drilled concrete tapered piles in cohesive-frictional soil. *Can. Geotech. J.* **2008**, *45*, 377–392. [CrossRef]
48. Li, H.; He, C.; Gong, Q.M.; Zhou, S.H.; Li, X.X.; Zou, C. TLM-CFSPML for 3D dynamic responses of a layered transversely isotropic half-space. *Comput. Geotech.* **2024**, *168*, 106131. [CrossRef]
49. He, C.; Zhou, S.H.; Di, H.G.; Guo, P.J.; Xiao, J.H. Analytical method for calculation of ground vibration from a tunnel embedded in a multi-layered half-space. *Comput. Geotech.* **2018**, *99*, 149–164. [CrossRef]