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A Homotopy Continuation Approach for Testing a Basic Analog Circuit

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Abstract

The increase of complexity on integrated circuits has also raised the demand for new testing methodologies capable to detect functional failures within circuits before they reach the market. Hence, this work proposes to explore the use of homotopy as a tool for testing a basic analog circuit. The homotopy path is influenced by nonlinearities from the equilibrium equation of the circuit; this situation can be used to infer faults by detecting changes on the homotopy path. The concept was explored using numerical simulation of a simple test circuit; then comparing results for the circuit with and without faults, obtaining modifications on the homotopy path like: the final point, number of iterations, and the number of turning points.

Keywords: *Homotopy continuation, circuit testing, analog circuits.*

1 Introduction

The vertiginous increase in the number of transistors per integrated circuit and increase of nonlinearities, present in the circuit, due to the decrease in the physical dimensions of the transistors, makes the integrated circuit testing area a challenge for engineers and scientists. The cost of testing an integrated circuit can represent, in average, the 50% for the total production cost, or even in some cases (specific circuits) represents up to 70% [1,2,3,4]. The development of strategies for integrated circuits testing; especially analog circuits or mixed signal, still is an open problem. These kinds of tests must assure complete functionality, quality and performance

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criterion, for each functional block, and the correct operation for the complete system, as well. Analog circuits are characterized by nonlinear characteristics, noise, bandwidth, and a wide variety of performance parameters. All of this becomes a constraint when developing fault simulators and reliable testing algorithms.

The homotopy continuation methods have been applied to various branches of science and engineering [5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22]. In [23] was presented a verification method of diagnosis for analog piecewise linear circuits based on the homotopy approach; here, the circuit is tested by homotopy transforming it from the fault-less state into a fault state. Thus, by monitoring electric variables, it is possible to detect value changes between states. Nevertheless, this testing approach requires PWL modelling of the circuit and ignores the entire behaviour of the homotopy path; thus, only cares about the nominal operating points between states.

Homotopy paths tend to be attracted by the traces formed by the intersection of equations of the equilibrium equation [24]. Hence, if faults directly affect the nonlinearities of the equilibrium equation, therefore, the homotopy path is influenced by nonlinearities from the equilibrium equation; this situation can be used to infer faults detecting behaviour changes on the homotopy path. Therefore, this work will present a study on how the homotopy path is affected by faults during DC analysis in parameters like: final point, number of iterations, and the number of turning points obtained for a basic multi stable [25,26,27] circuit in fault-less and fault state.

This paper is organized as follows. In Section 3, we present the basic idea of proposed double bounded homotopy. Section 4 shows some numerical simulations to study the effects of faults over homotopy trajectories and discuss the results. Finally, a brief conclusion is given in Section 5.

2 Basis of Homotopy Continuation Methods

The homotopy continuation methods (HCM) are a continuous transformation from one trivial problem (simple to solve) to the study problem (hard to solve). These kind of methods are applied to such diverse problems like: multi-stable electronic circuits [16,28,29,30,31,32,33,34,35,36,37,38], Toeplitz systems, nonlinear control synthesis [39], stochastic economies [40], load flow solutions of ill-conditioned power systems [41], discretization of ordinary differential equations [42], inverse kinematics problems [20,43,44,45,46,47,48,49], optimization [50,51], among many others.

First, we define our problem to solve, the nonlinear algebraic equation

$$f(x) = 0, \qquad f: \in \Re^n \longrightarrow \Re^n, \quad (2.1)$$

where x denotes the variables of the problem and n is the total number of those variables.

Then, in order to solve (2.1), we propose the following homotopy map [19,21,22,28,29,31,32,46,50,51,52,53,54,55,56,57,58,59,60,61] as

$$H(f(x), \lambda) = 0, \qquad H: \in \Re^{n+1} \longrightarrow \Re^n, \quad (2.2)$$

where λ is the homotopy parameter.
Equation (2.2) represents any homotopy formulation that fulfils the following conditions:

- For $\lambda = 0$, solution for $H^{-1}(0)$ is known or easily found using numerical methods.
- For $\lambda = 1, H(f(x), 1) = f(x)$. It means that at $\lambda = 1$ the solution, or solutions, for $f(x)$ can be found.
- The path for $H^{-1}(0)$ is a continuous function of λ with $0 \leq \lambda \leq 1$.

The homotopy path is the solution set for $H^{-1}(0)$, which represents a continuous curve that can be traced by numerical continuation techniques or path following methods [5,16,21,33,47,48,62,63,64].

A possible homotopy map is

$$H(f(x), \lambda) = \lambda f(x) + (1 - \lambda) g(x) = 0, \tag{2.3}$$

where $g(x)$ is a problem simple to solve.
If $\lambda = 0$, the homotopy path is reduced to the trivial problem

$$H(x, 0) = g(x) = 0. \tag{2.4}$$

When $\lambda = 1$, the sought solution is achieved

$$H(x, 1) = f(x) = 0. \tag{2.5}$$

This process is a continuous deformation from $\lambda = 0$ to $\lambda = 1$, transforming the trivial problem $g(x) = 0$ into the original problem $f(x) = 0$.

The success of finding the sought solution depends on several factors:

- Find the methodology to establish the right NAEs which best describe the physical behaviour of the problem under study. The proper formulation of those NAEs can help to guarantee, or increase the probability, of success of the homotopy simulation [cite].
- The behaviour of different homotopy maps change for specific problems.
- Constructing an adequate numerical continuation algorithm. Even if the homotopy path and NAEs are properly established with the guarantee of global convergence [31,65], a poorly chosen numerical continuation scheme can lead to a failure [5,16,21,33,62,64].

Homotopy is capable to find multiple solutions; nevertheless, the ability to find all or even one of the solutions depends on the kind of homotopy, the selected continuation technique [16,33], and on the type of nonlinear circuit. To be able to find multiple solutions, all the steps for the numerical continuation method [5,16,33] must be applied until the root at $\lambda = 1$ is located. Nonetheless, the tracing technique continues its route for values $\lambda > 1$ up to a turning point and then returns back to $\lambda = 1$ finding the next solution (when another solution exist). In Fig. 1 a homotopy path is shown; here, the path locates solutions at x_1^*, x_2^*, x_3^*, and x_4^*. However, after crossing the root x_4^* the path continues indefinitely as apparently there are no more solutions that produce more crossings at $\lambda = 1$.

3 Proposed DBPH Method for Analog Circuit Testing

In [28,29] was introduced the idea to create a double bounded homotopy; a way to cope with the stop criterion is by bending the open solution path to convert it into a closed path as depicted in Fig. 2. This is achieved by setting four solution lines in terms of a fixed separation d as shown in Fig. 2. Further properties of this scheme are given as follows:

- At $\lambda_i = d/2$ the symmetry axis is defined (starting point A).
- For $\lambda_i$the solution for $H^{-1}(0)$is known or computationally simple to obtain. This point is known as the initial point for the homotopy (λ_i, x_i).
- At $\lambda = 0$, the homotopy formulation becomes

$$H(f(x),0) = f(x) = 0.$$

This means that at $\lambda = 0$ all solutionsfor $f(x)$ arelocated.

- Similarly, at $\lambda = d$, thehomotopy fulfils

$$H(f(x), d) = f(x) = 0,$$

which means that at $\lambda = d$ all solutionsfor $f(x)$ arelocated.

- The expressions above define two symmetric branches; the left branch for $0 \leq \lambda \leq \lambda_i$ and the right branch for $\lambda_i \leq \lambda \leq d$.
- When a given branch reaches again the value λ_i (final point B), the homotopy procedure stops, achieving a simple stop criterion. In Fig. 2, the path starts at point A and stops at point B.
- The path for the inverse function$H^{-1}(0)$ is a continuous function forλinthe range of $0 \leq \lambda \leq d$.

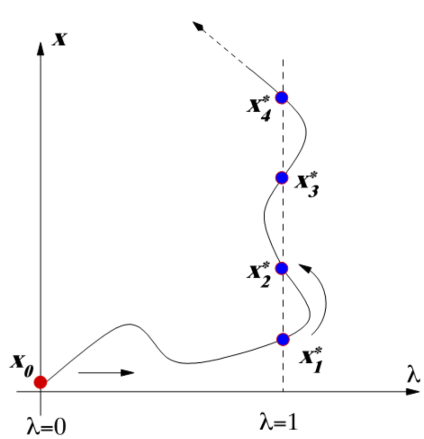

Fig. 1. Stop criterion problem

Not only the properties shown above yield a simple and reliable stop criterion, but they also establish two solution lines that, infact, limit the swing of the homotopy parameter variation. Fig. 2 shows how the homotopy path starts at $A = (d/2, x_i)$ on the symmetry axis, finds two roots(in solutionline $\lambda = d$) and finishes when a new crossing through the symmetry axis at $B = (d/2, x_f)$ is detected; it means that tracing the symmetrical branch has been completed and the stop criterionhasbeen fulfilled.

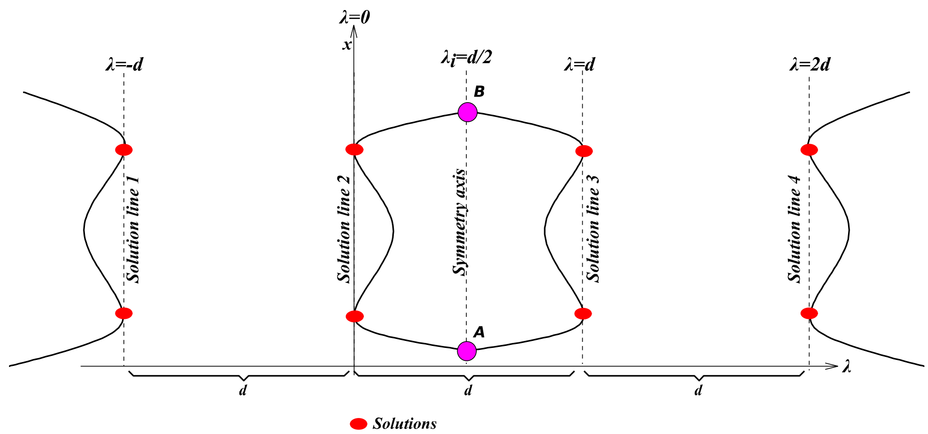

Fig. 2. Double bounded homotopy with four solution lines

The DBPH homotopy method [29] is a kind of double bounded homotopy, which is proposed to infer the existence of faults

$$f(x) = (\lambda + d)\lambda(\lambda - d)(\lambda - 2d)(x - x_i)(x - x_f) - c\left(\lambda - \frac{d}{2}\right)^2 f(x)^2 \qquad (3.1)$$

Where λ is the homotopy parameter,$f(x)$ is the equilibrium equation for the circuit,$d$is a constant that represents separation between solution lines ($\lambda = -d$, $\lambda = 0$, $\lambda = d$, and $\lambda = 2d$)[29],x_i represents theinitialpoint, x_f represents the final point of the path at $\lambda = d/2$ (symmetryaxis), and c is another constant.

This method is characterized by creating an arbitrary closed path around the symmetry axis; this situation allows for an arbitrary set of starting points for the path.

A fault or fabrication defect can be ideally modelled as the unexpected existence of spurious resistors between nodes in the circuit. It may happen, as an example, the appearance of a spurious resistor between the *i*-th node and ground. This will affect, directly, the equilibrium equation, in particular the nodal equationf_i. As smooth as this change may appear, it could unchain a series of changesin the shape of the homotopy path. This way, it is possible to correlate an alteration in the homotopy path with the presence of faults.

In this work we use the Euler-predictor and Newton-corrector scheme described in [16], which is based on other reports from [5,64], to obtain the numerical simulations.

4 Numerical Simulation and Discussion

The benchmark circuit with bipolar transistors in Fig. 3 was reported and solved using HCM methods [29,30]. The Ebers-Moll [31] model isused for all the transistors; and the equation for the model is given as

$$\begin{bmatrix} i_E \\ i_C \end{bmatrix} = \begin{bmatrix} 1 & -0.01 \\ -0.99 & 1 \end{bmatrix} \begin{bmatrix} 10^{-9}\left(e^{(40v_{be})} - 1\right) \\ 10^{-9}\left(e^{(40v_{bc})} - 1\right) \end{bmatrix},$$

where i_E represents the emitter current, i_C represents the collector current, v_{be} is the voltage drop betweenbase and emitter, and v_{bc} is the voltage drop between base and collector.

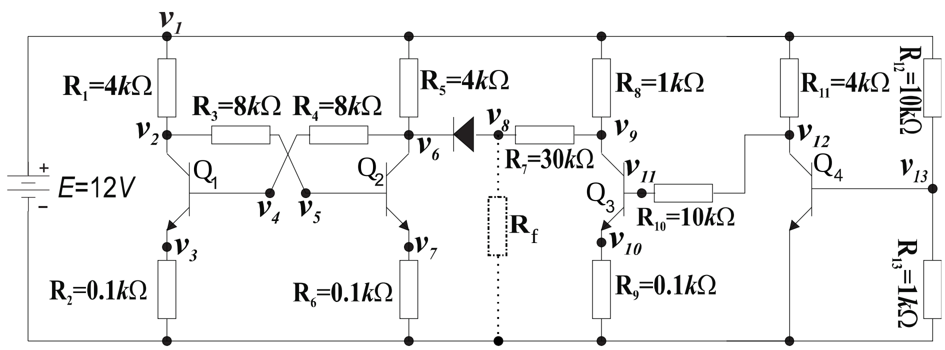

Fig. 3. Circuit under test (contains three operating points). $R_f = 11k\Omega$ models a possible fault between node v_8 and ground

Table 1. Initial point ($x_A$and x_B) and final points ($x_{f_{1x}}$ and $x_{f_{2x}}$) for the paths. b) Solutions for the circuit with (S_{xF}) and without fault (S_x)

(a)

x	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9	v_{10}	v_{11-13}	$I_{V_{CC}}$	T.P.	Iter.
x_A	+	-	+	-	-	-	-	-	-	+	-	+		
$x_{f_{1A}}$	+	+	+	+	+	+	+	+	+	+	+	-	5	15522
$x_{f_{2A}}$	+	+	+	+	-	+	+	+	+	+	+	-	5	12753
x_B	-	-	+	-	-	-	-	-	-	+	-	+		
$x_{f_{1B}}$	+	-	+	-	-	-	+	-	+	+	-	-	11	28532
$x_{f_{2B}}$	+	+	+	+	-	+	-	-	-	+	-	-	15	26726

(b)							
R.P.	v_1	v_2	v_3	v_4	v_5	v_6	v_7
S_1	12	0.405	0.366	0.685	0.349	6.796	0.070
S_{1F}	12	0.40237	0.36085	0.68080	0.34736	6.40881	0.06886
S_2	12	0.883	0.278	0.590	0.631	0.812	0.315
S_{2F}	12	0.86554	0.27808	0.59147	0.62122	0.81387	0.30548
S_3	12	5.995	0.085	0.368	0.712	0.436	0.390
S_{3F}	12	6.06730	0.08213	0.36504	0.70724	0.43067	0.38564
R.P.	v_8	v_9	v_{10}	v_{11}	v_{12}	v_{13}	I_E
S_1	7.038	11.839	0.4E-5	0.039	0.039	0.321	-0.0085
S_{1F}	3.14292	11.7142	0.3E-5	0.03884	0.03874	0.32149	-0.00874
S_2	1.074	11.647	0.4E-5	0.039	0.039	0.321	-0.0100
S_{2F}	1.06759	11.6472	0.3E-5	0.03884	0.03874	0.32149	-0.01009
S_3	0.699	11.635	0.4E-5	0.039	0.039	0.321	-0.0089
S_{3F}	0.68858	11.6350	0.3E-5	0.03884	0.03874	0.32149	-0.00889

As for the diode, the model is given by

$$i_d = 10^{-9}(e^{40u} - 1),$$

where u is the voltage drop between diode terminals and id is the current through the diode.

First, the equilibrium equation for the fault-free circuit is formulated using the modified nodal analysis [32]; the result is a system having 14 equations and 14 variables

f_1) $(1.85\text{E}-3)v_1 - (2.5\text{E}-4)v_2 - (2.5\text{E}-4)v_6 - (1\text{E}-3)v_9 - (2.5\text{E}-4)v_{12} - (1\text{E}-4)v_{13} + i_E = 0,$

f_2) $-(2.5\text{E}-4)v_1 + (3.75\text{E}-4)v_2 - (1.25\text{E}-4)v_5 + (9.9\text{E}-9)\exp(40v_4 - 40v_3) + (1\text{E}-10) - (1\text{E}-8)\exp(40v_4 - 40v_2) = 0,$

f_3) $(1\text{E}-2)v_3 - (1\text{E}-8)\exp(40v_4 - 40v_3) + (9.9\text{E}-9) + (1\text{E}-10)\exp(40v_4 - 40v_2) = 0,$

f_4) $(1.25\text{E}-4)v_4 - (1.25)v_6 + (1\text{E}-10)\exp(40v_4 - 40v_3) - (1\text{E}-8) + (9.9\text{E}-9)\exp(40v_4 - 40v_2) = 0,$

f_5) $-(1.25\text{E}-4)v_2 + (1.25\text{E}-4)v_5 + (1\text{E}-10)\exp(40v_5 - 40v_7) - (1\text{E}-8) + (9\text{E}-9)\exp(40v_5 - 40v_6) = 0,$

f_6) $-(2.5\text{E}-4)v_1 - (1.25\text{E}-4)v_4 + (3.75\text{E}-4)v_6 + (9.9\text{E}-9)\exp(40v_5 - 40v_7) + (1.01\text{E}-8) - (1\text{E}-8)\exp(40v_5 - 40v_6) - (1\text{E}-8)\exp(40v_8 - 40v_6) = 0,$ (4.1)

f_7) $(1\text{E}-2)v_7 - (1\text{E}-8)\exp(40v_5 - 40v_7) + (9.9\text{E}-9) + (1\text{E}-10)\exp(40v_5 - 40v_6) = 0,$

f_8) $(30\text{E}3)^{-1}v_8 - (30\text{E}3)^{-1}v_9 + (1\text{E}-8)\exp(40v_8 - 40v_6) - (1\text{E}-8) = 0,$

f_9) $-(1\text{E}-3)v_1 - (30\text{E}3)^{-1}v_8 + (31)(30\text{E}3)^{-1}v_9 + (9.9\text{E}-9)\exp(40v_{11} - 40v_{10}) + (1\text{E}-10) - (1\text{E}-8)\exp(40v_{11} - 40v_9) = 0,$

f_{10}) $(1\text{E}-2)v_{10} - (1\text{E}-8)\exp(40v_{11} - 40v_{10}) + (9.9\text{E}-9) + (1\text{E}-10)\exp(40v_{11} - 40v_9) = 0,$

f_{11}) $(1\text{E}-4)v_{11} - (1\text{E}-4)v_{12} + (1\text{E}-10)\exp(40v_{11} - 40v_{10}) - (1\text{E}-8) + (9.9\text{E}-9)\exp(40v_{11} - 40v_9) = 0,$

f_{12}) $-(2.5\text{E}-4)v_1 - (1\text{E}-4)v_{11} + (3.5\text{E}-4)v_{12} + (9.9\text{E}-9)\exp(40v_{13}) +$

$$(1\mathrm{E}-10)-(1\mathrm{E}-8)\exp(40v_{13}-40v_{12})=0,$$
$f_{13})$ $$-(1\mathrm{E}-4)v_1+(1.1\mathrm{E}-3)v_{13}+(1\mathrm{E}-10)\exp(40v_{13})-(1\mathrm{E}-8)+ (9.9\mathrm{E}-9)\exp(40v_{13}-40v_{12})=0,$$
$f_{14})$ $$v_1-12=0.$$

Fig. 3 shows a hypothetical resistor ($R_f=11k\Omega$) representing a fault between node 8 and ground; acting as a currentleak to the ground. The existence of the fault implies that, for the circuit in a fault state, this term willbe added

$$+\frac{1}{R_f}v_8$$

to nodal equation f_8 within equilibrium equation (4.1).

Initial point for DBPH homotopy is chosen at ±13V, for nodal values, and ±13A for the current$I_{V_{CC}}$of voltage source V_{CC}. Initial points x_A and x_B are shown in Table 1(a), just marking the corresponding sign (plus sign means +13, while minus sign means -13; the units would be Volts or Amperes depending whether is voltage or current, respectively).

Now, DBPH homotopy is applied to solve the circuit; the proposed homotopy formulation is expressed as follows

$$\begin{aligned}
H_1)\quad & (\lambda_1+1)\lambda_1(\lambda_1-1)(\lambda_1-2)(v_1-13)(v_1+13)+(\lambda_1-0.5)^2f_1^2=0,\\
H_2)\quad & (\lambda_1+1)\lambda_1(\lambda_1-1)(\lambda_1-2)(v_2-13)(v_2+13)+(\lambda_1-0.5)^2f_2^2=0,\\
& \vdots\\
H_{13})\quad & (\lambda_1+1)\lambda_1(\lambda_1-1)(\lambda_1-2)(v_{13}-13)(v_{13}+13)+(\lambda_1-0.5)^2f_{13}^2=0,\\
H_{14})\quad & (\lambda_1+1)\lambda_1(\lambda_1-1)(\lambda_1-2)(I_E-13)(I_E+13)+(\lambda_1-0.5)^2f_{14}^2=0,
\end{aligned} \tag{4.2}$$

where, by simplification, parameters d and C are both set to 1.

After numerical simulation there are some aspects of the results to be noticed:

1. Two sets of simulations (A and B) were performed, using two different initial points (x_A and x_B) for the circuits with and without fault (Table 1(a) and Figs. 4 and 5). For both cases, differences on the final point of the path were detected. For instance, between final points $x_{f_{1A}}$(non-fault circuit) and $x_{f_{2A}}$(fault circuit) there is a difference on the final point for nodal voltagev_5. Also, for final points $x_{f_{1B}}$(non-fault circuit) and $x_{f_{2B}}$(fault circuit) a noticeable differenceis detected because sign changes for the final nodal voltages v_2, v_4, v_6, v_7, and v_9. Therefore, a fault can modify the final point of the homotopy path; this situation can be used as criteria to detect faults.
2. For all cases (circuits with and without fault) three operating points were located (Table 1(b)). Therefore, the number of located roots may not necessarily be used as an indicator or criterion of the existence of faults. Nevertheless, between solutions $S_1$and S_{1F}a noticeablechange on nodal voltage v_8 is perceived because the fault is located, precisely, between v_8 and ground.
3. The shape of the homotopy path for the circuit without fault (Fig. 4(a), (b) and Fig. 5(a), (b)), differs with respect to the homotopy path for a fault circuit (Fig. 4(c), (d) and Fig. 5(c), (d)). The change with respect to the homotopy may be quantified by monitoring the number of iteration steps for the numerical continuation method [28,33] and the number of turning

points as a reference or indicator for fault detection. In Table 1(a) can be seen that the number of iterations and the number of turning points vary for the homotopy simulation between the circuits with and without fault. This situation confirms the existence of differences in the nonlinearities of the equilibrium equation for circuits with and without fault; differences that produce, in the end, changes on the homotopy paths.

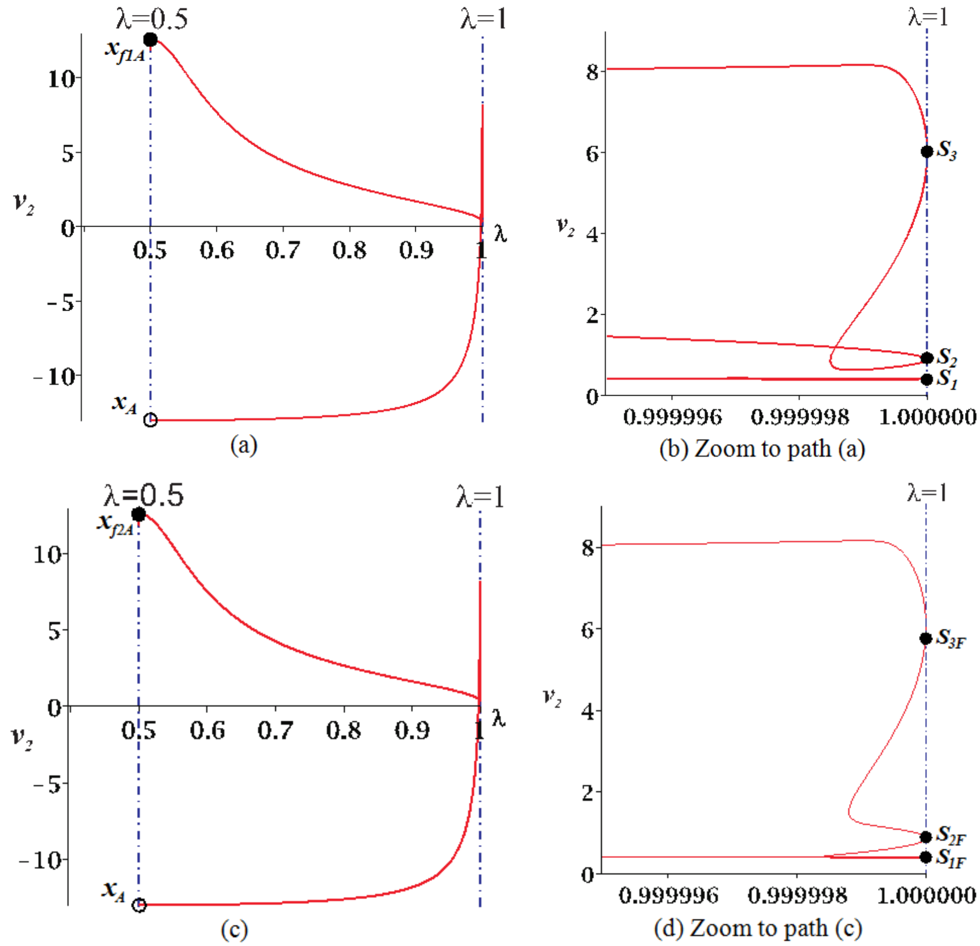

Fig. 4. Homotopy paths $\lambda - v_2$ for initial point at x_A: (a), (b) the fault-free circuit and (c), (d) fault circuit

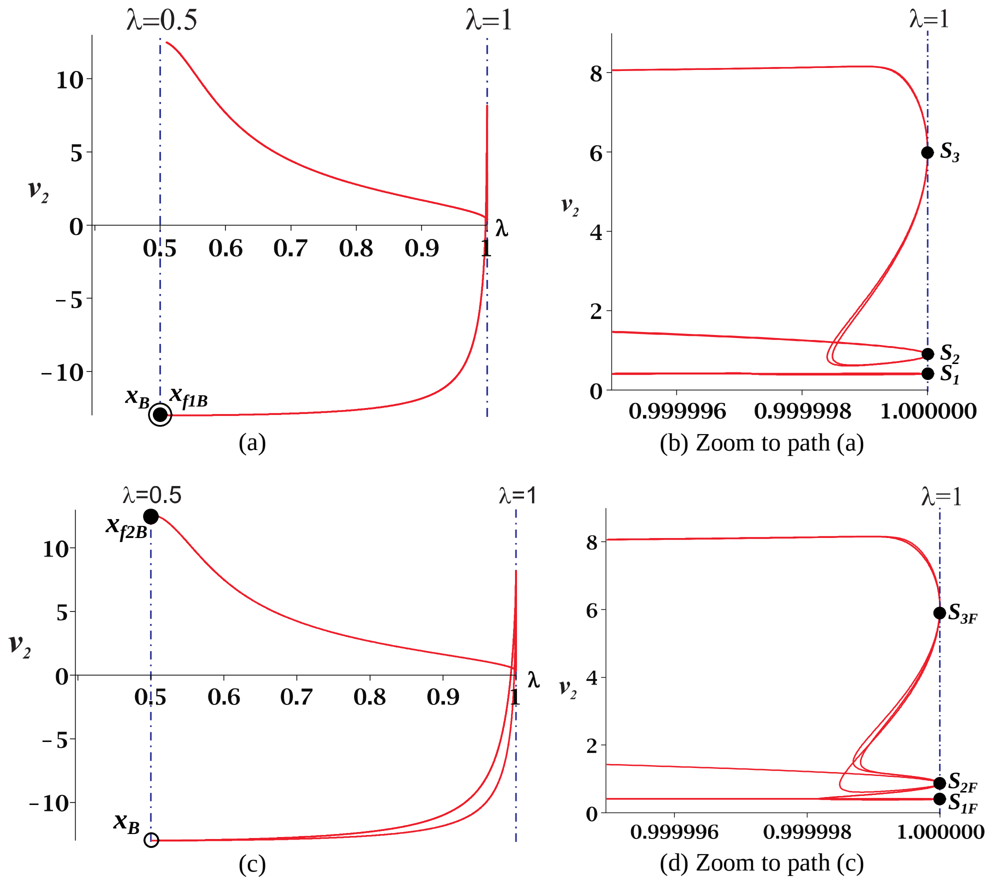

Fig. 5. Homotopy paths $\lambda - v_2$ for initial point at x_B: (a), (b) the fault-free circuit and (c), (d) fault circuit

This work exhibited that aspects like final point for the homotopy path; number of turning points, and number of iterations can reflect the existence of circuit faults. Also, the present study can be extended to use other homotopies like Newton homotopy [33], fixed point [30], among others, in order to determine which homotopy is the most sensitive to faults. In addition, future research may extend the application of the proposed study to practical switching circuits and, in particular, fault models [66,67].

5 Conclusion

This work deals with the application of homotopy to the analog circuit testing area, proposing the feasibility to detect the presence of faults using the homotopy path analysis, involving turning points, number of iterations, and final point of the path. Results indicate that the aforementioned aspects undergo notorious changes comparing circuit simulations with and without fault.

Therefore, it is important to continue the research in order to implement a systematic criterion to locate faults.

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Competing Interests

Authors have declared that no competing interests exist.

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